Control of Sleep:

Theory and Experiments

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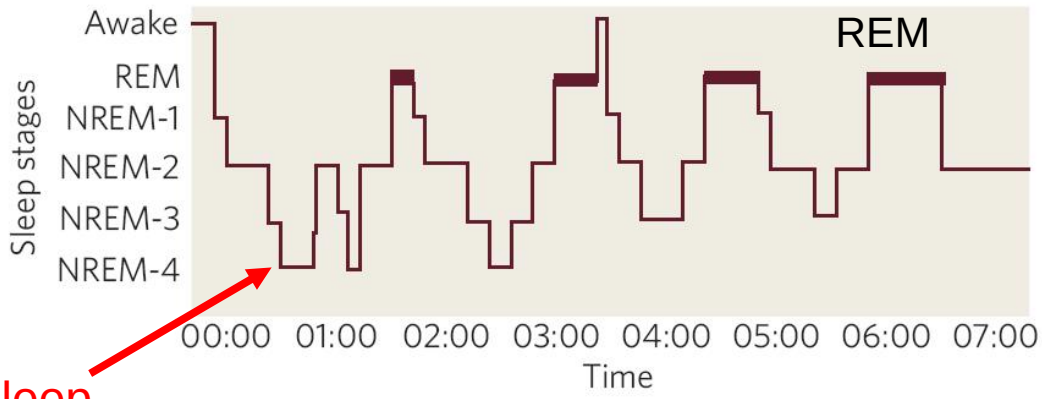
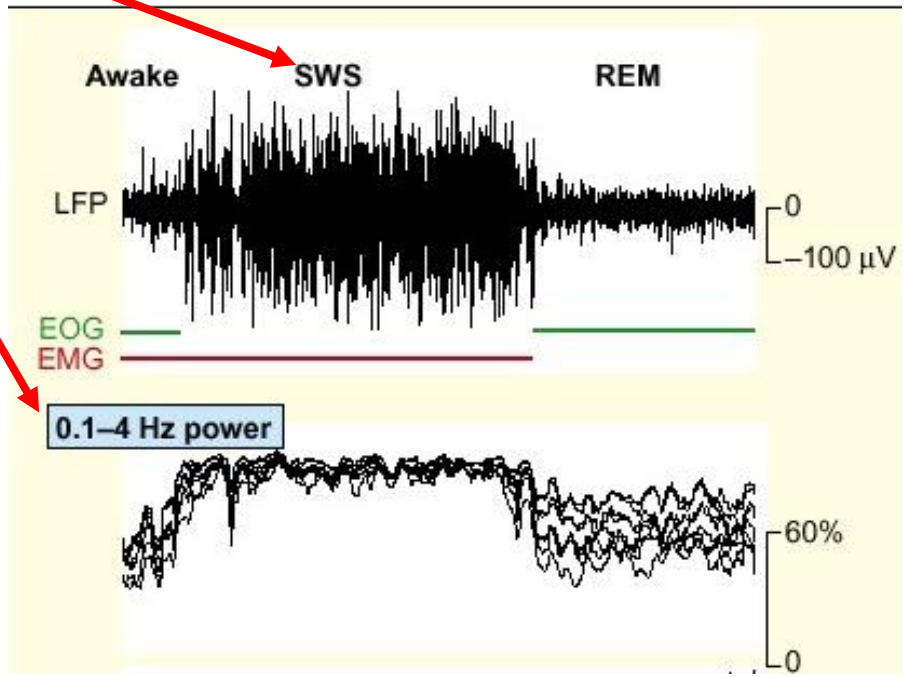
You sleep about 1/3 of your life

How ? Why? Control?

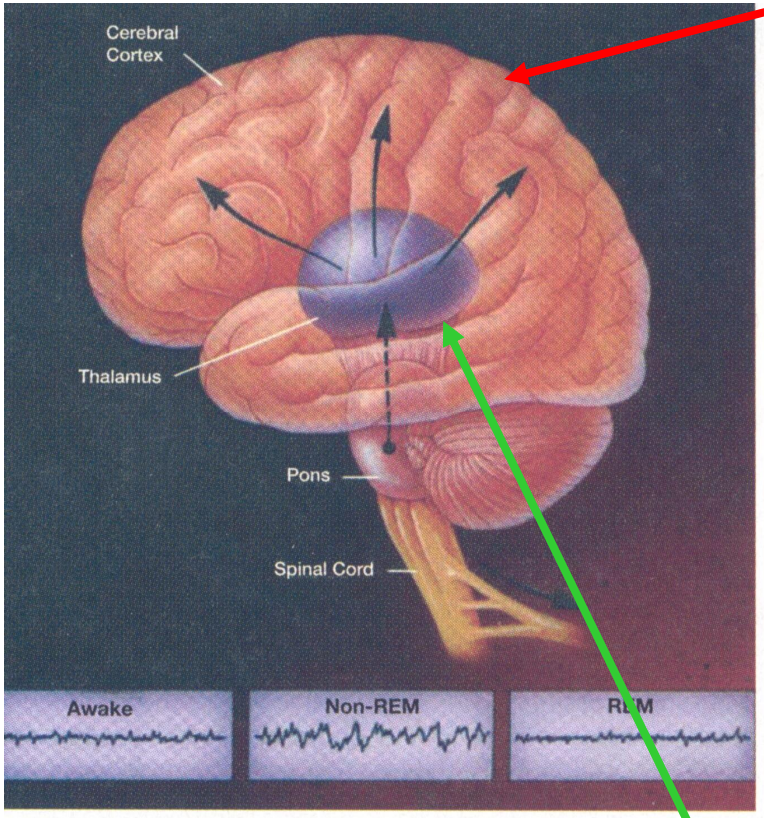
Sleep Stages

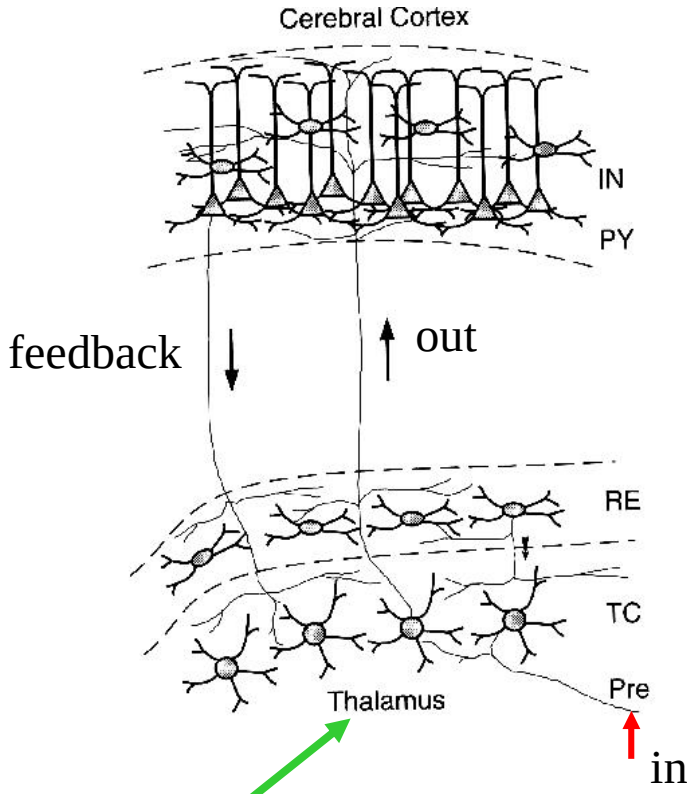


Slow Wave Sleep



Slow waves are generated in the Cortex

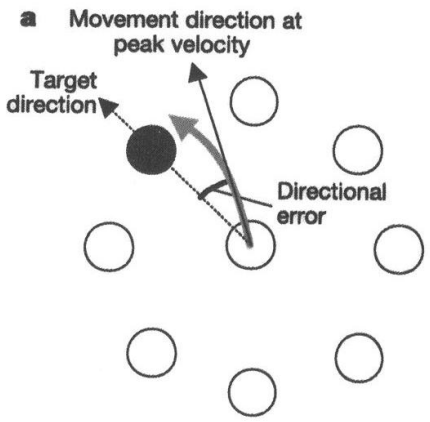




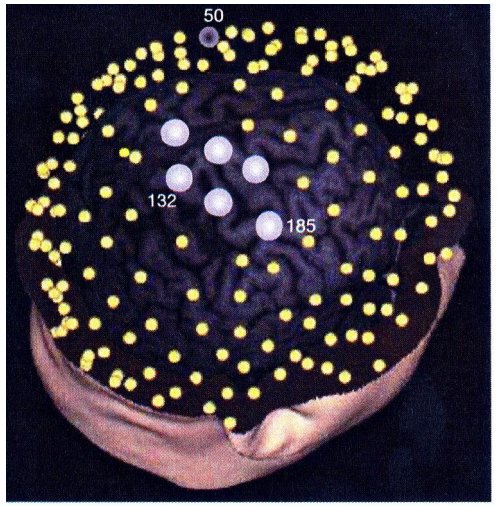
Slow waves synchronize the Thalamus = Doorway to the Brain

Slow wave sleep consolidates memory

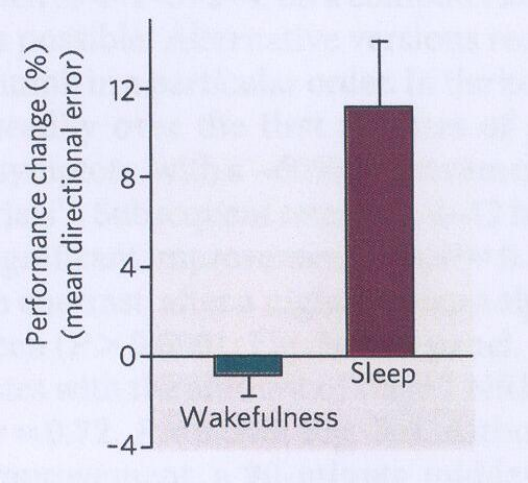
Error CompensationTask



Local Slow Wave Sleep

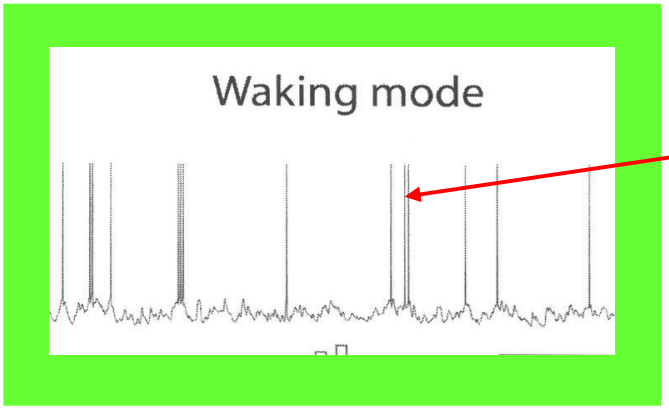


Improvement of performance after sleep

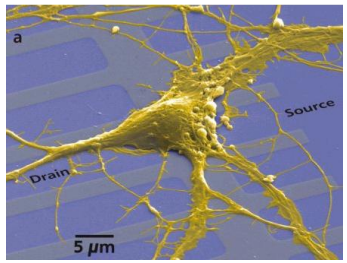


Huber et al. Nature **430** 78 (2004)

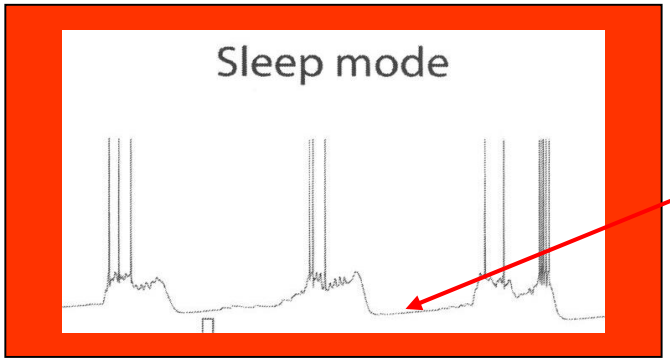
Timescales



Neural Spiking



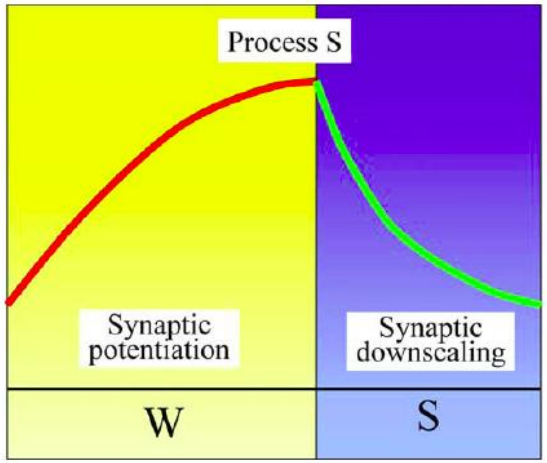
Milliseconds



Switch off

(Hyperpolarization)

Seconds



Synaptic changes

Homeostasis

Consolidation of memory

Minutes, Hours

How we will proceed

1. Model for Slow Waves

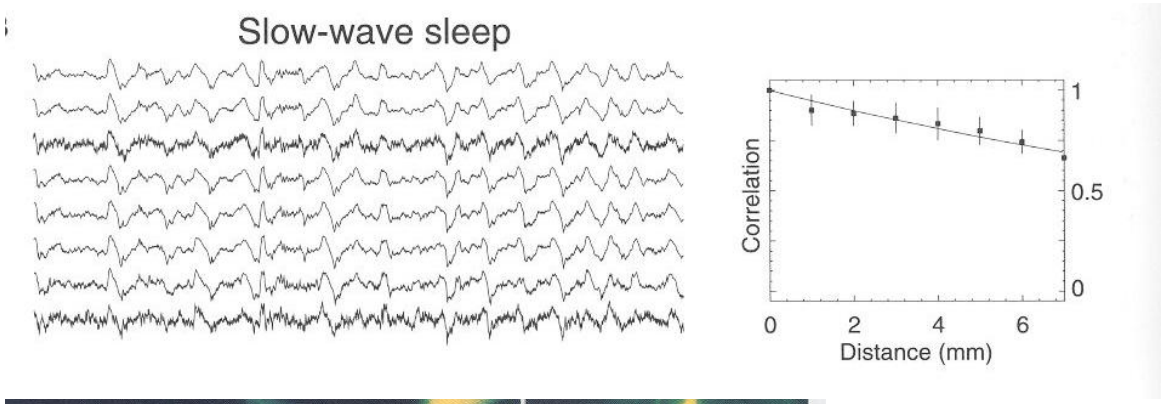
2. Control via Inhibition

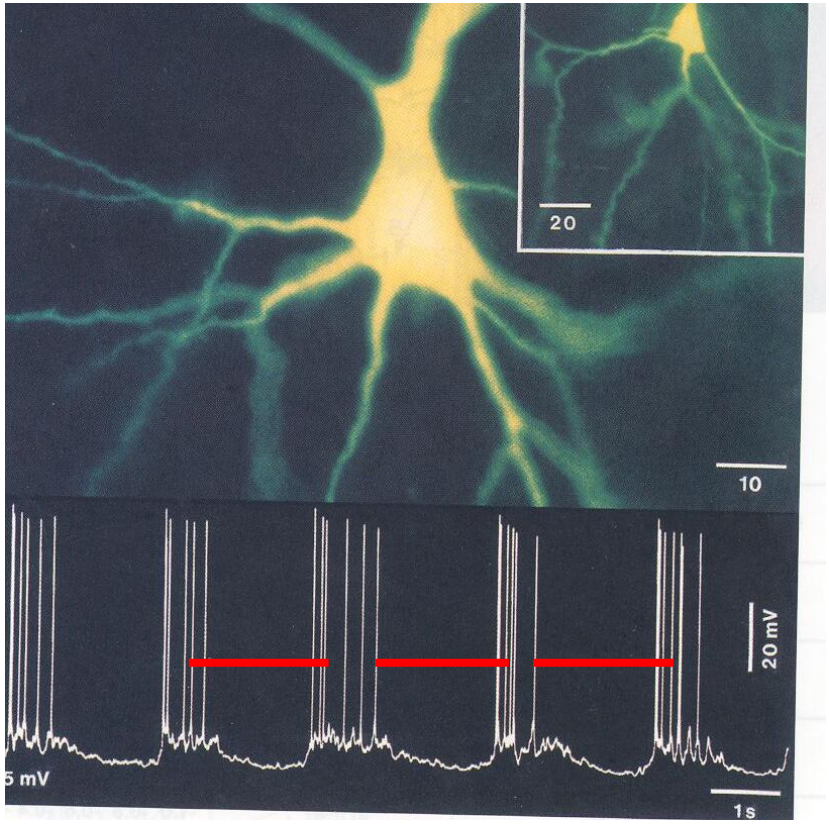
3. Transition to Synchrony

4. Control via Synaptic Change

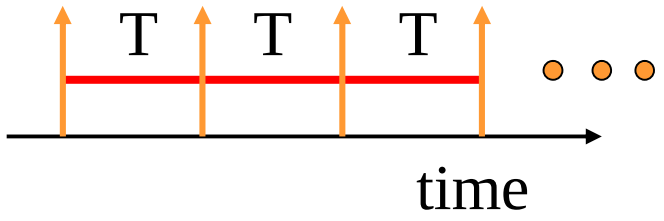
Model for Slow Waves

Synchronized activity





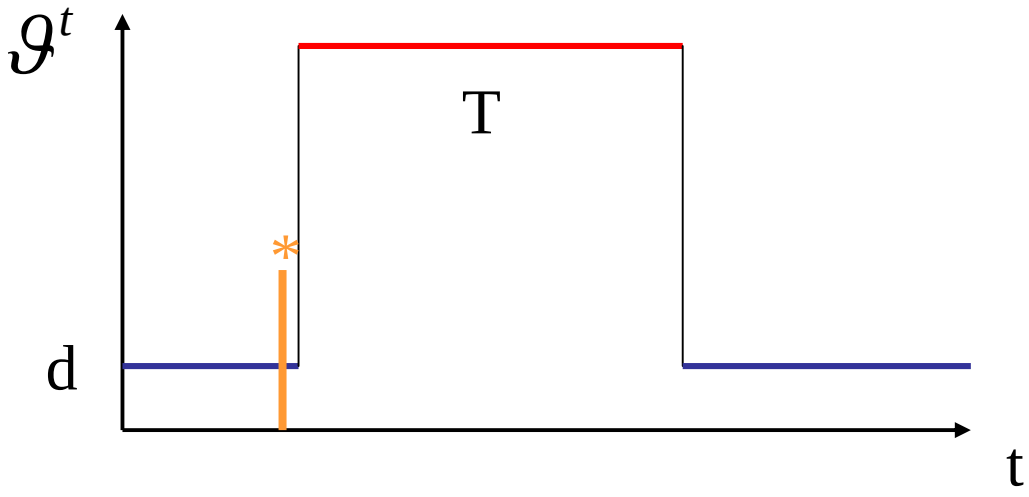
Model



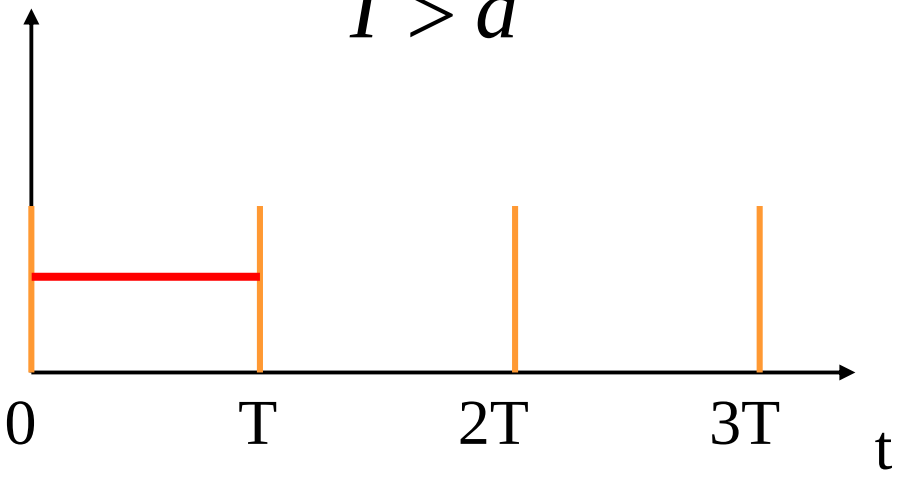
Single Threshold Element with Selfinhibition

$$x^{t+1} = \theta\left(I^t + \xi^t - \vartheta^t\right)$$

Output 0,1 — Input — Noise — Threshold

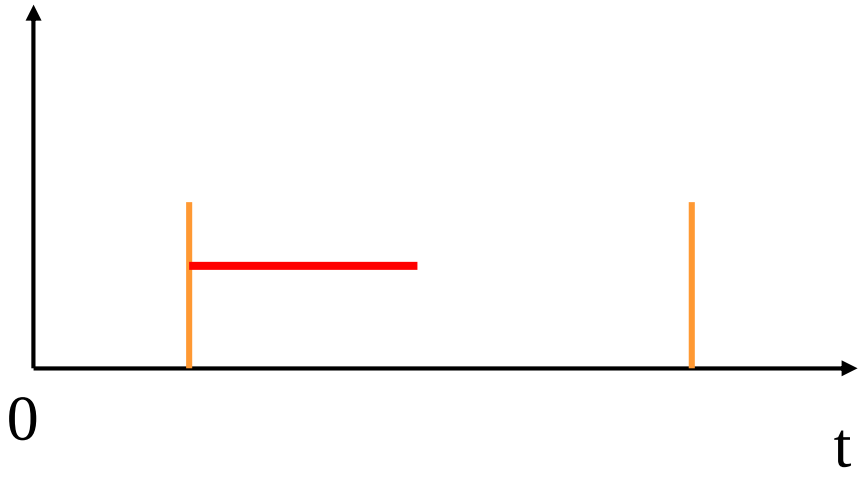


$I > d$



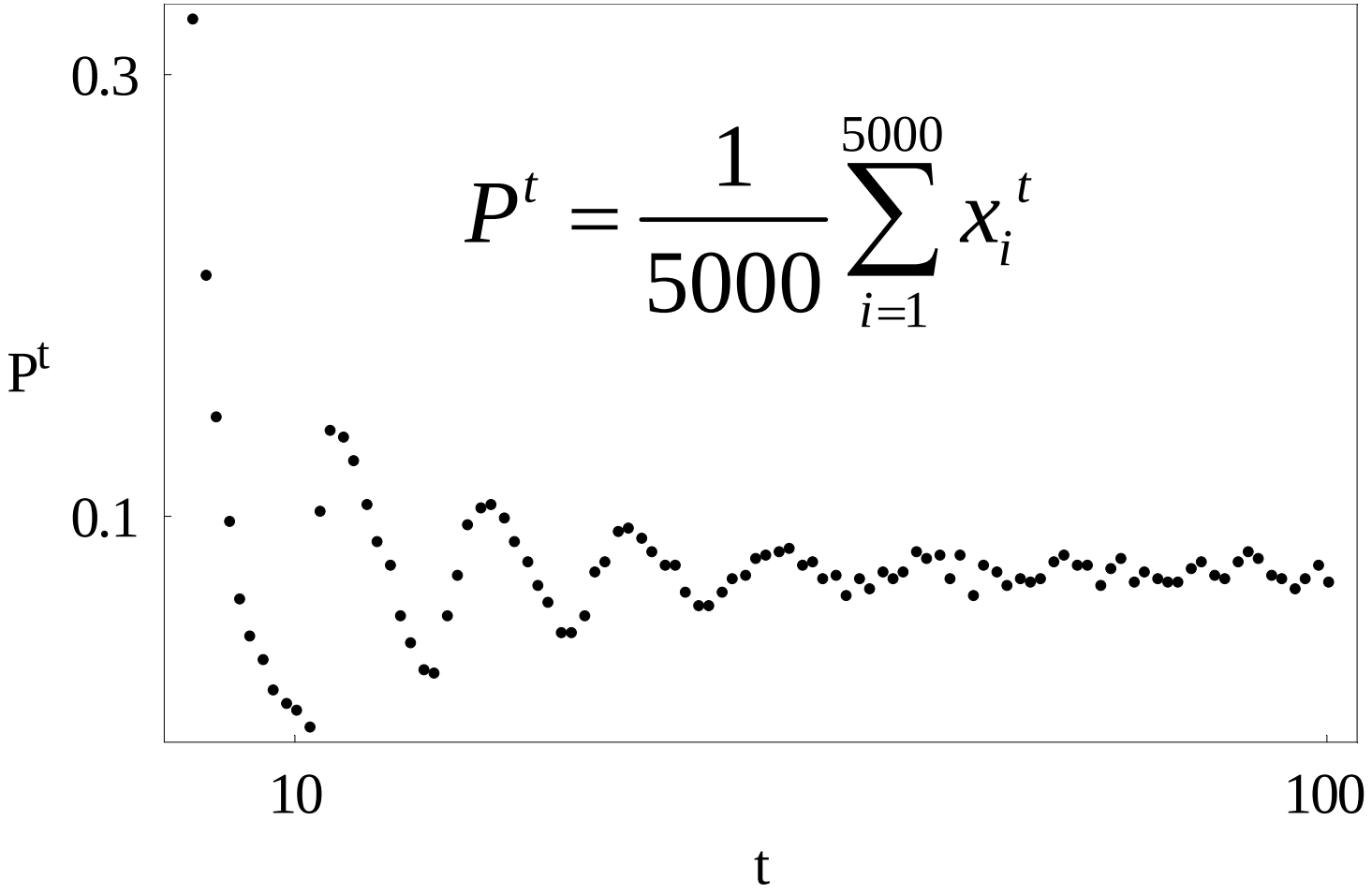
Without noise but constant input I

$I = 0 \qquad \xi^t \neq 0$

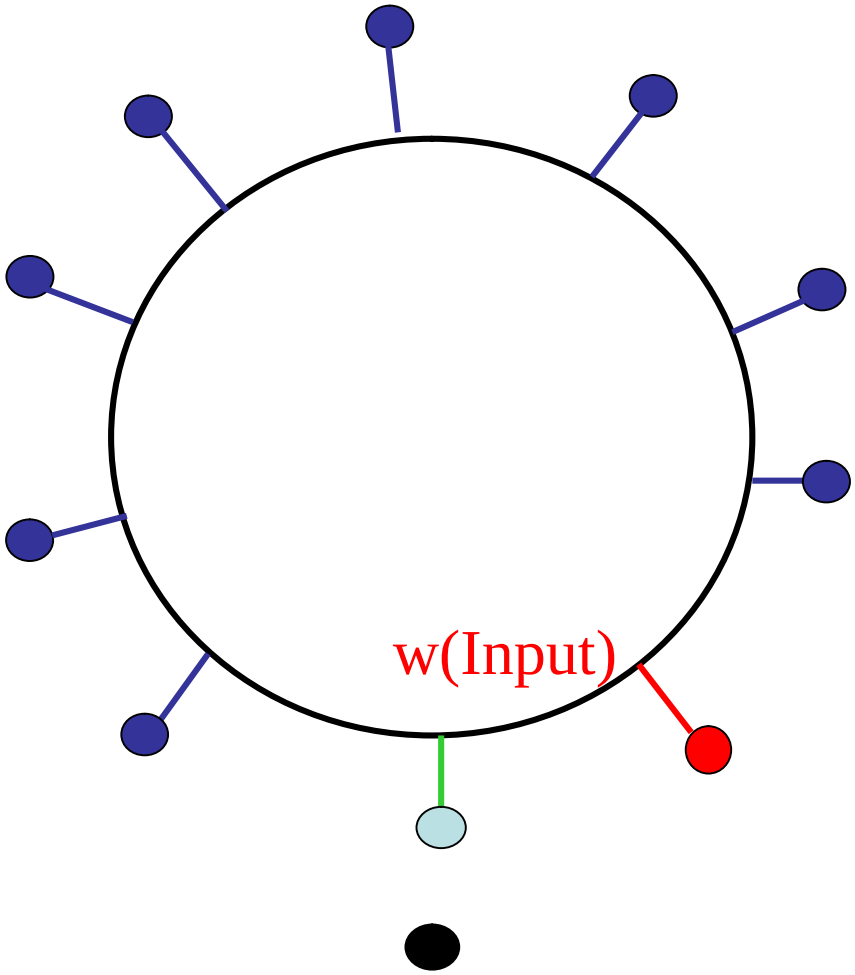


With weak noise

Average over 5000 Elements with Gaussian Noise, T=10



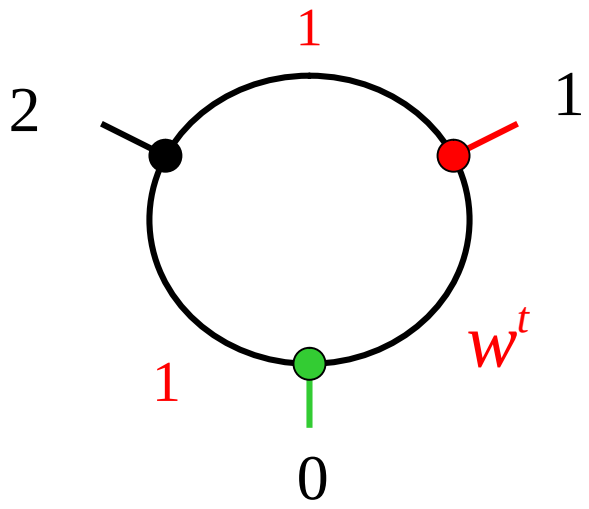
Probabilistic
Automaton



dead

active

excitable



$$P_0^{t+1} = P^t{}_0 - w^t P_0^t + P_2^t$$

$$P_1^{t+1} = P_1^t + w^t P_0^t - P_1^t = w^t P_0^t$$

$$P_2^{t+1} = P_2^t + P_1^t - P_2^t = P_1^t$$

$$P_1^{t+1} = w^t P_0^t$$

$$= w^t \left(1 - P_1^t - P_2^t\right) \qquad P_0^t + P_1^t + P_2^t = 1$$

$$= w^t \left(1 - P_1^t - P_1^{t-1}\right) \qquad P_2^{t+1} = P_1^t \rightarrow P_2^t = P_1^{t-1}$$

$$P^{t+1} = w^t \left(1 - \sum_{\tau=0}^{T_0-1} P^{t-\tau}\right)$$

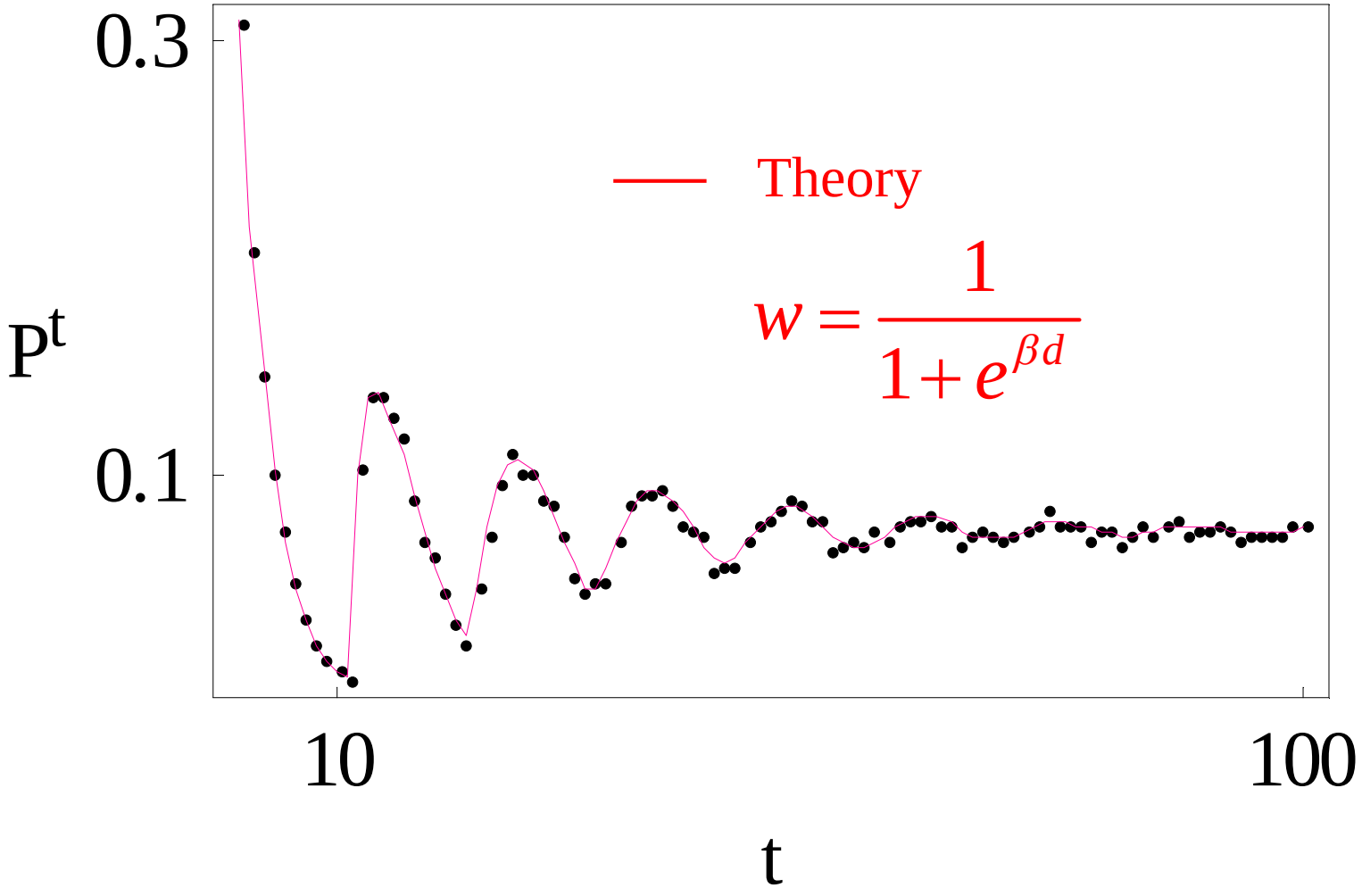
Probability of firing

Time dependence of the probability P^t of firing

$$P^{t+1} = w^t \left(1 - \sum_{\tau=0}^{T-1} P^{t-\tau}\right)$$

$$w^t = \left\langle \theta\left(I^t + \xi - d\right)\right\rangle_{\text{Noise}}$$

$$w^t = \frac{1}{1 + e^{-\beta\left(I^t - d\right)}} \qquad \beta = \sqrt{2}/\sigma$$

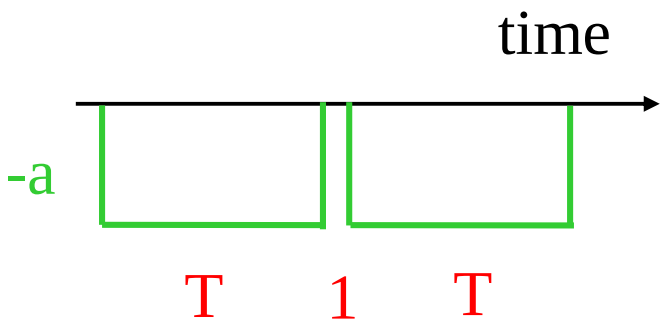

0.3
0.1
P^t
Theory
$w = \frac{1}{1+e^{\beta d}}$
10
100
t

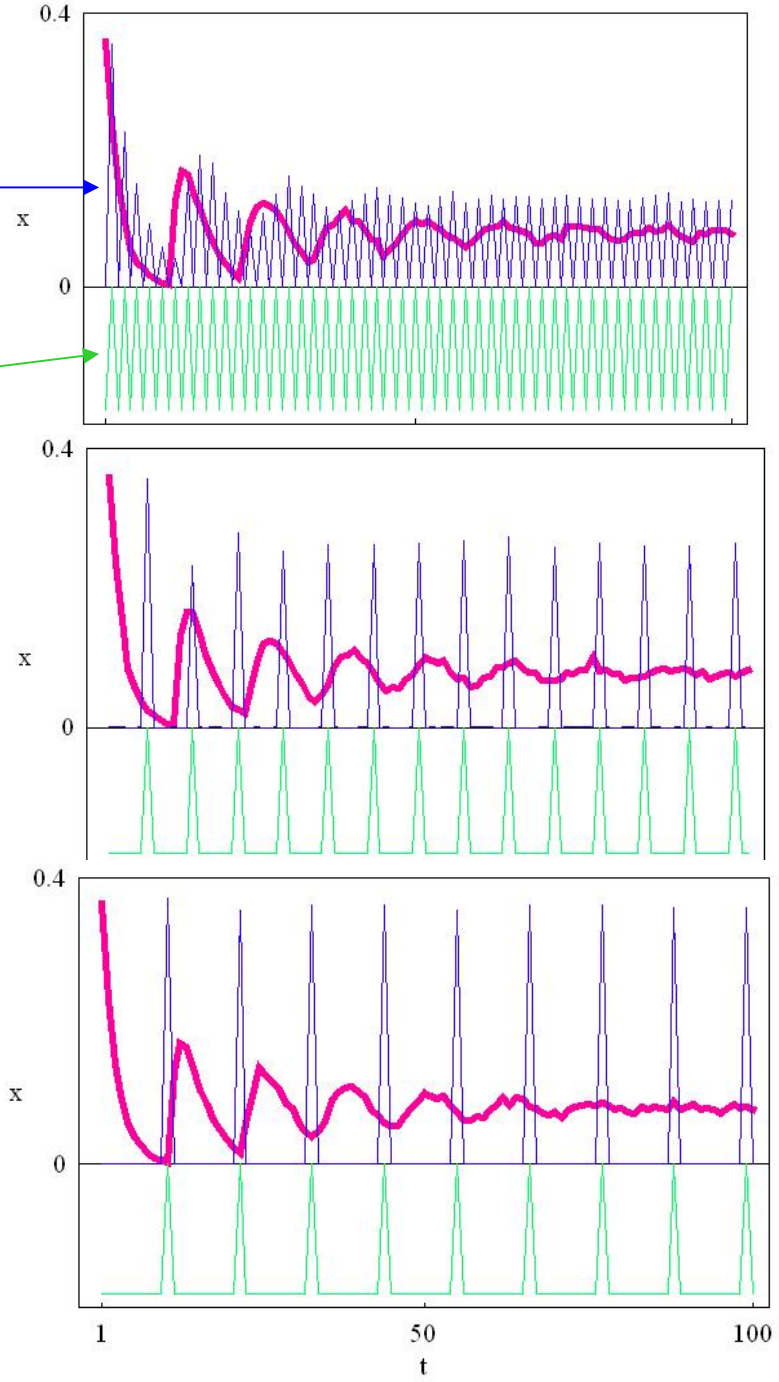
Control via Inhibition

Driven System

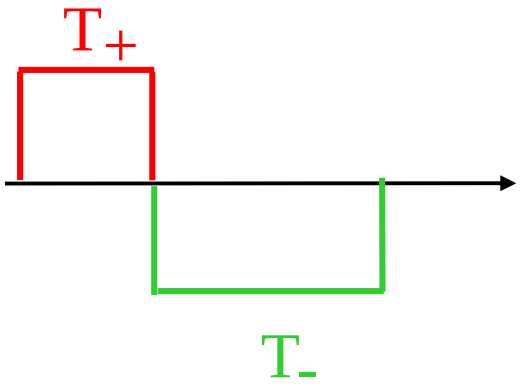
Response

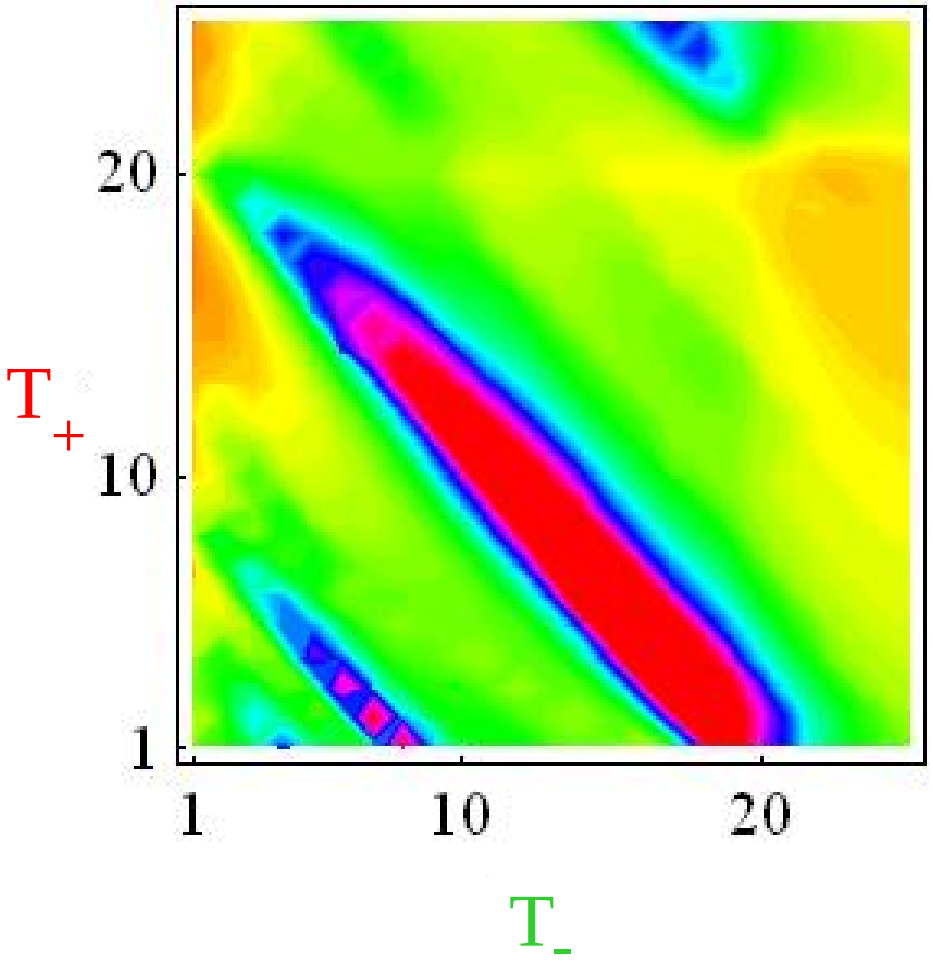
External
inhibiting signal

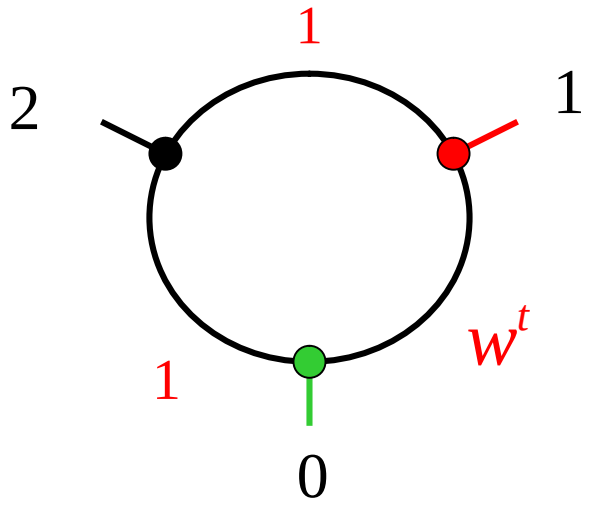




External
Signal







$$P_0^{t+1} = P^t{}_0 - w^t P_0^{\,t} + P_2^{\,t}$$

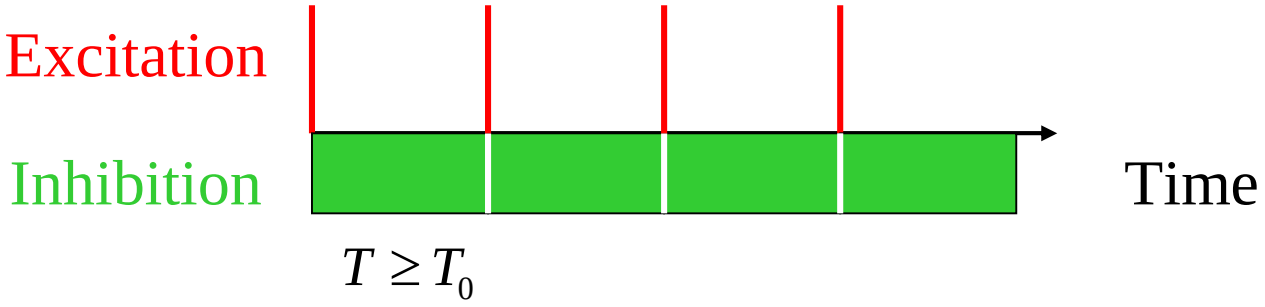
$$P_1^{t+1} = P_1^t + w^t P_0^{\,t} - P_1^t = w^t P_0^{\,t}$$

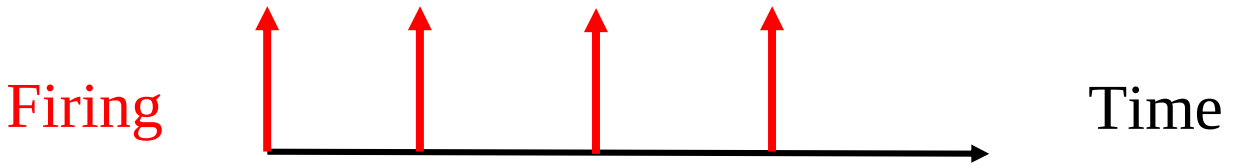
$$P_2^{t+1} = P_2^t + P_1^t - P_2^t = P_1^t$$

$$\begin{pmatrix} P_0^{t+1} \\ P_1^{t+1} \\ P_2^{t+1} \end{pmatrix} = \begin{pmatrix} 1-w^t & 0 & 1 \\ w^t & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} P_0^t \\ P_1^t \\ P_2^t \end{pmatrix}$$

$$\vec{P}^{t+1} = M(w^t)\vec{P}^t$$

System follows perfectly an external drive if perfectly inhibitory phases ($w = 0$) are long $T \geq T_0$

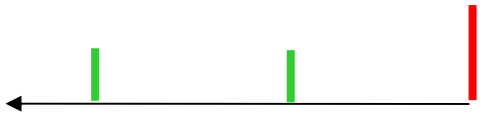




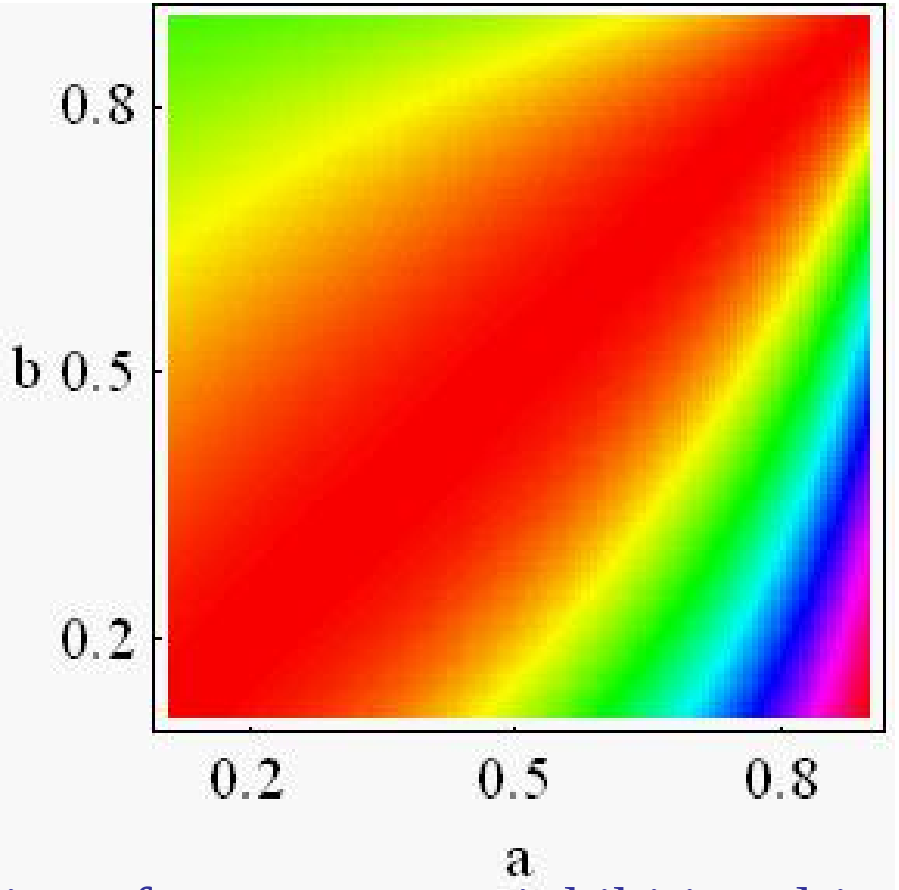
$$\vec{P}^{t+1} = M(w^t)\vec{P}^t \rightarrow M(0)^{T \geq T_0}\vec{P}^t = \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix} \qquad M(1)\begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \dots \end{pmatrix}$$

T=2

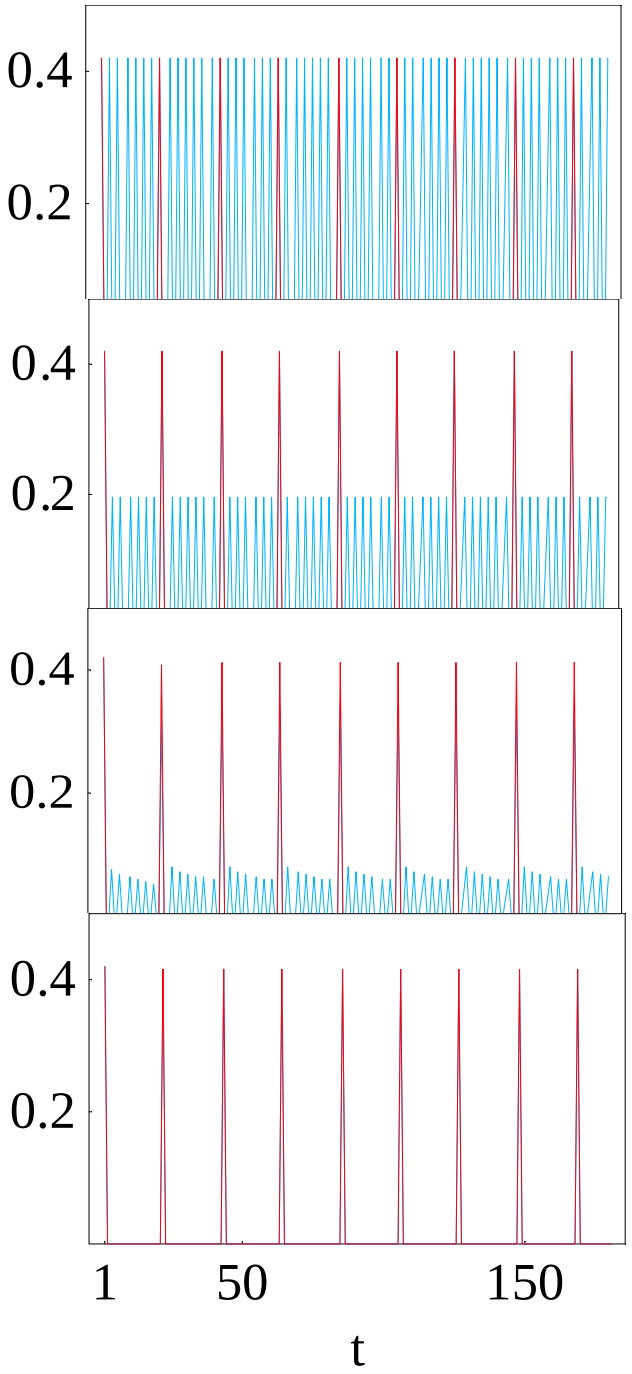
$$M(b)M(b)M(a)\vec{P} = \vec{P}$$



$$Variance = \frac{1}{3}\frac{(a-b)^2}{(1+b-2ab)^2}$$



Large spontaneous activity a favours strong inhibiting drive b→0

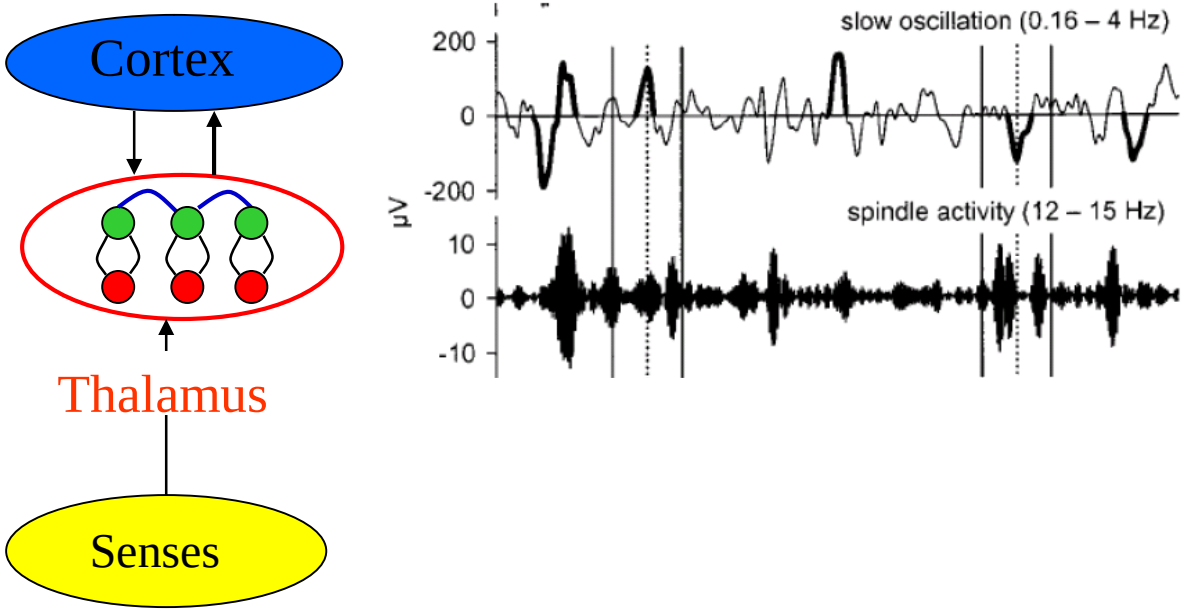


Two coupled ensembles with different frequencies

Increasing inhibitory coupling

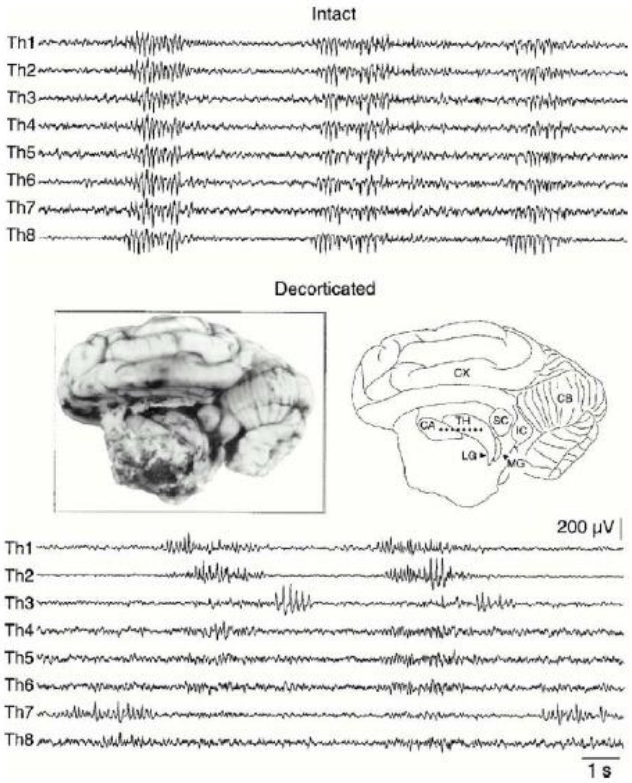
Slow enslaves Fast

Control of synchrony of thalamic spindles by corticothalamic projections



Mölle M, Marshall L, Gais S, Born J (2002) Grouping of spindle activity during slow oscillations in human non-REM sleep. J Neurosci 15: 10941-10947

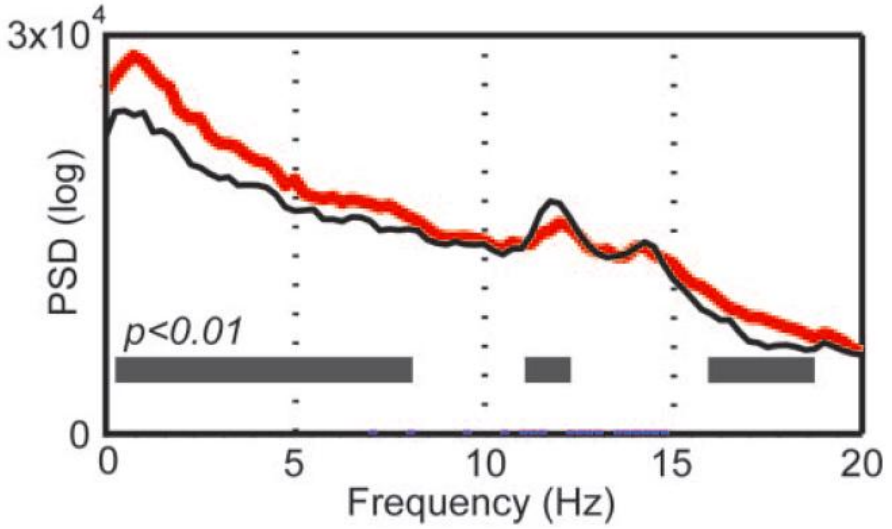
Corticothalamic projections control synchrony of thalamic spindles

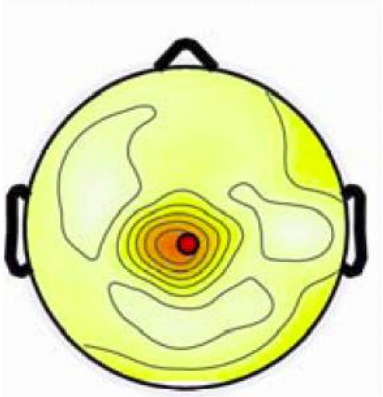


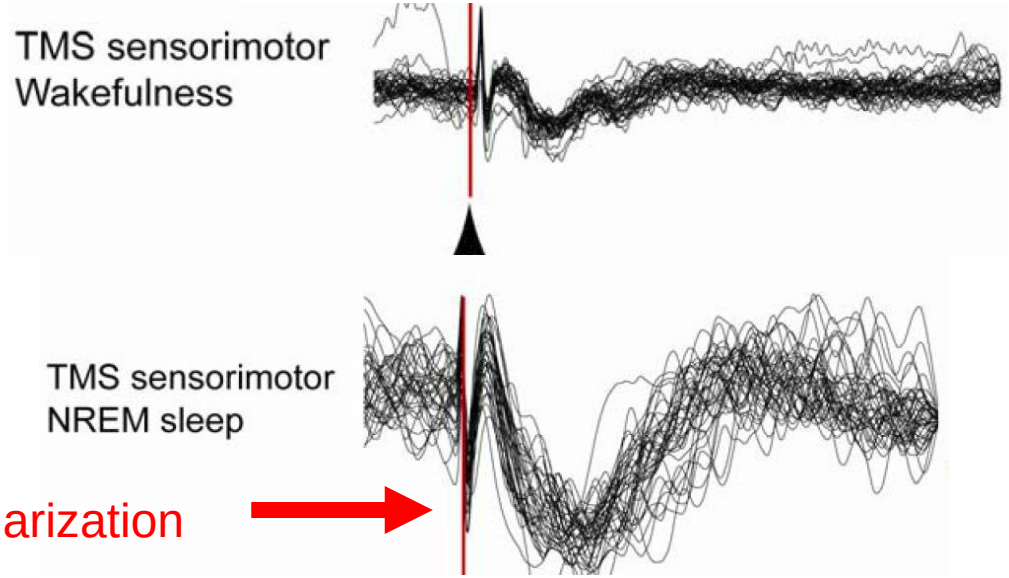
- in the intact brain spindles occur simultaneously over widespread territories of the thalamus
- after ablation of the neocortex synchrony of thalamic spindles is lost
- $\Longrightarrow$ corticothalamic projections control synchrony of thalamic spindles

Diego Contreras, Alain Destexhe, Terrence J. Sejnowski, Control of Spatiotemporal Coherence of a Thalamic Oscillation by Corticothalamic Feedback, Science **274**, 771, (1996)

Transcranial magnetic stimulation





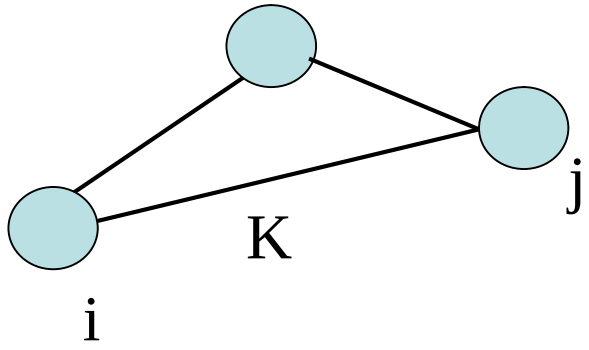


Depolarization

Massimi et al. PNAS 104 8496 (2007)

Transition to Synchrony

All to All Coupled Elements



$$w^t = \frac{1}{1+e^{\beta\left(I^t - d\right)}} \qquad \bar{P}^t = \frac{1}{N}\sum_{j=1}^{N} P_j^{\,t}$$

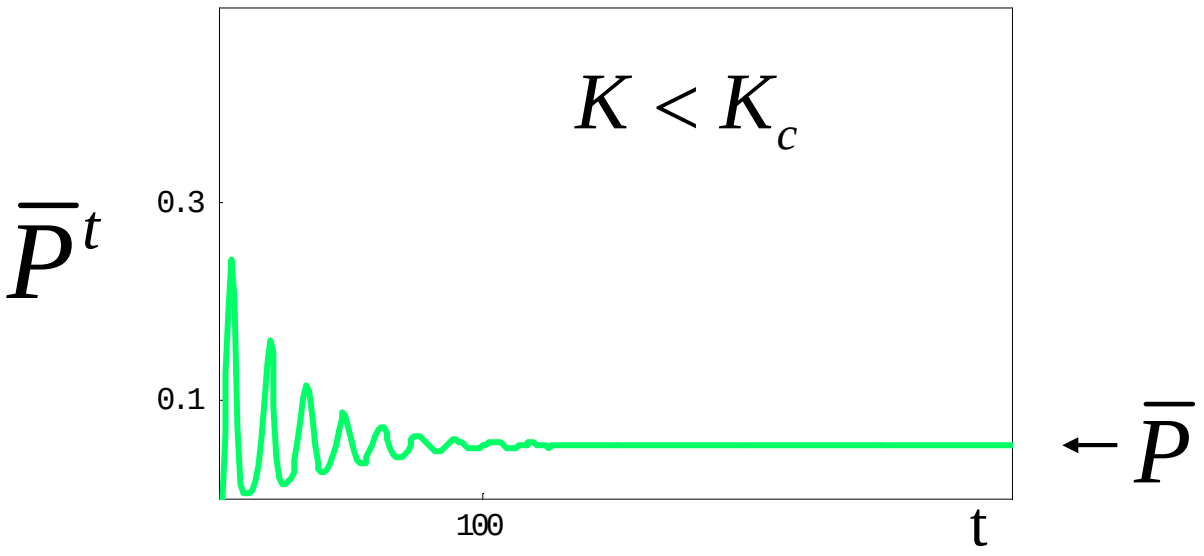
$$= \frac{1}{1+e^{\beta\left(K\bar{P}^t - d\right)}}$$

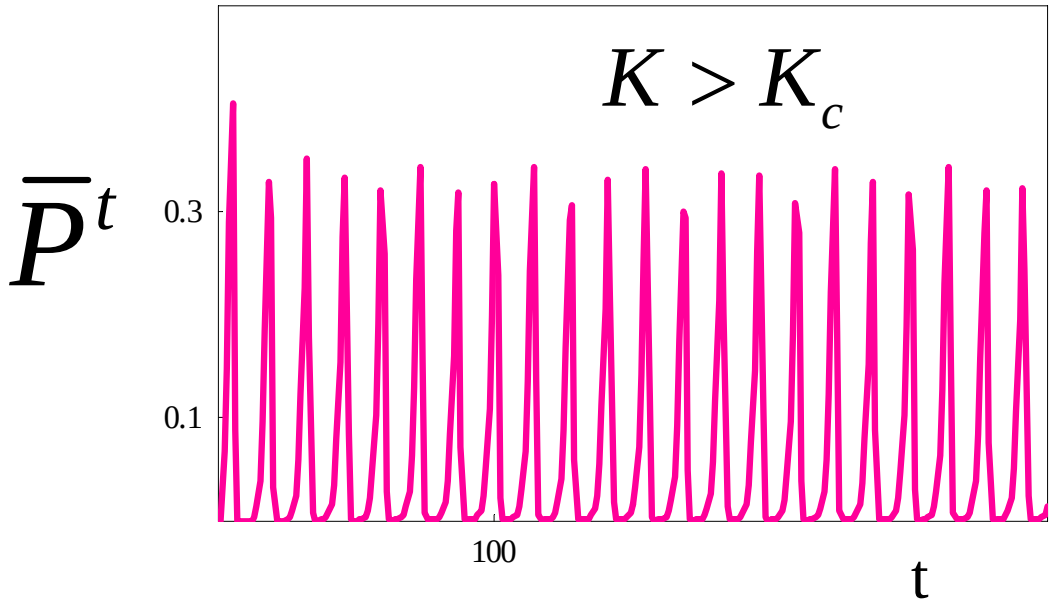
Mean Field Map

$$\bar{P}^{t+1} = w(K\bar{P}^{t})(1-\sum_{\tau=0}^{T}\bar{P}^{t-\tau})$$

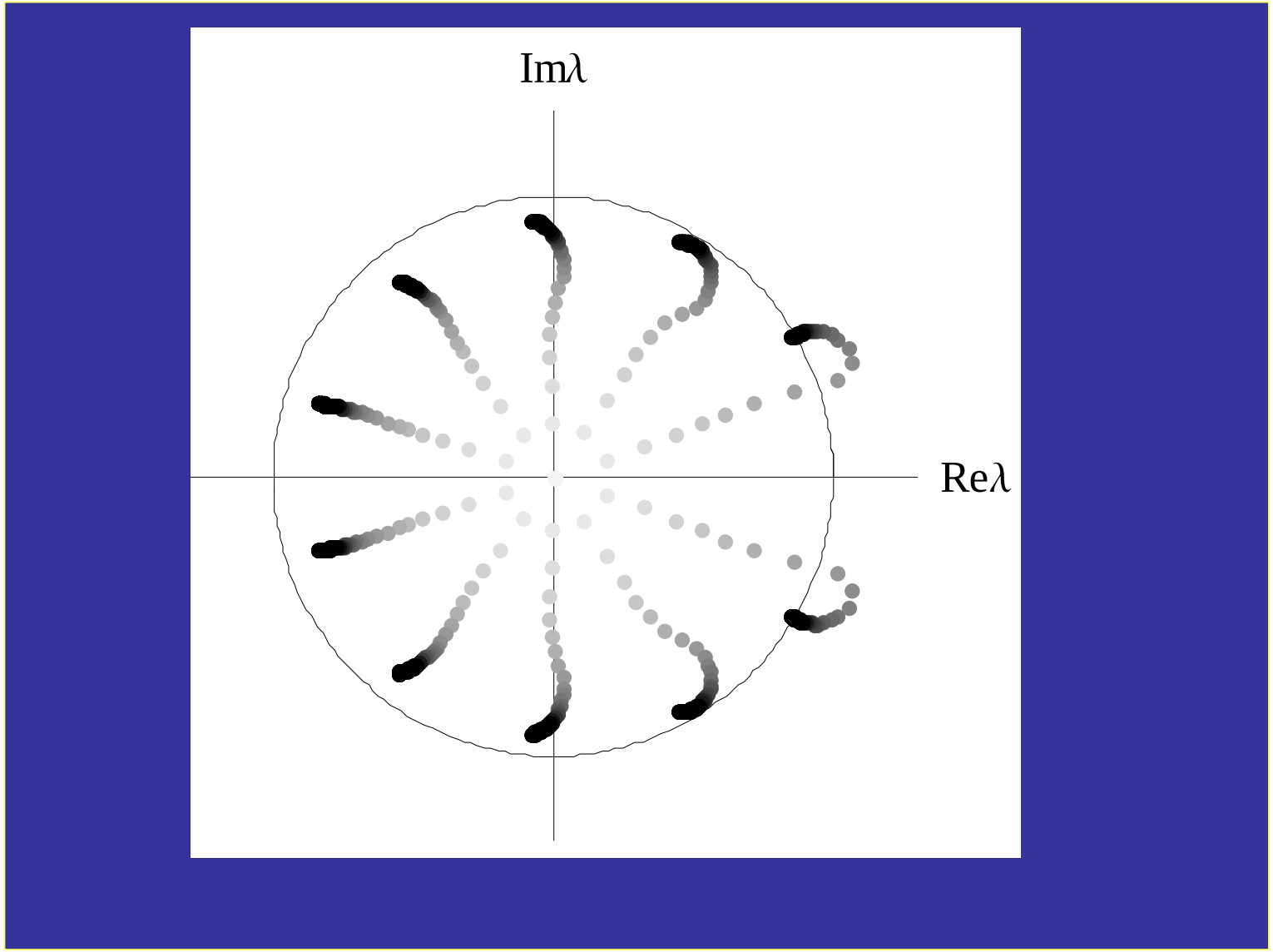
$$w(K\bar{P}^{t}) = \frac{1}{1+e^{-\beta\left(K\bar{P}^{t}-d\right)}}$$

$$P^{1} = \ldots\ldots = P^{T} = 0$$

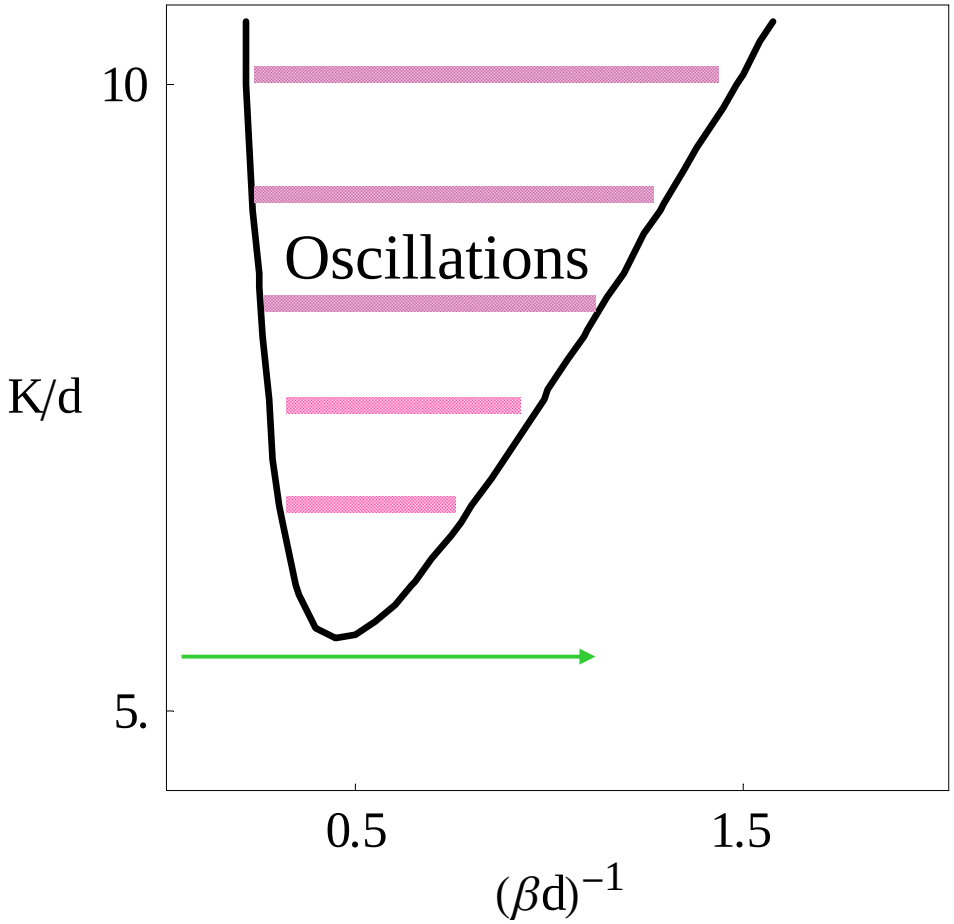

$K < K_c$
$\overline{P}^t$
0.3
0.1
100
t
$\leftarrow \overline{P}$


$K > K_c$
$\overline{P}^t$
0.3
0.1
100
t

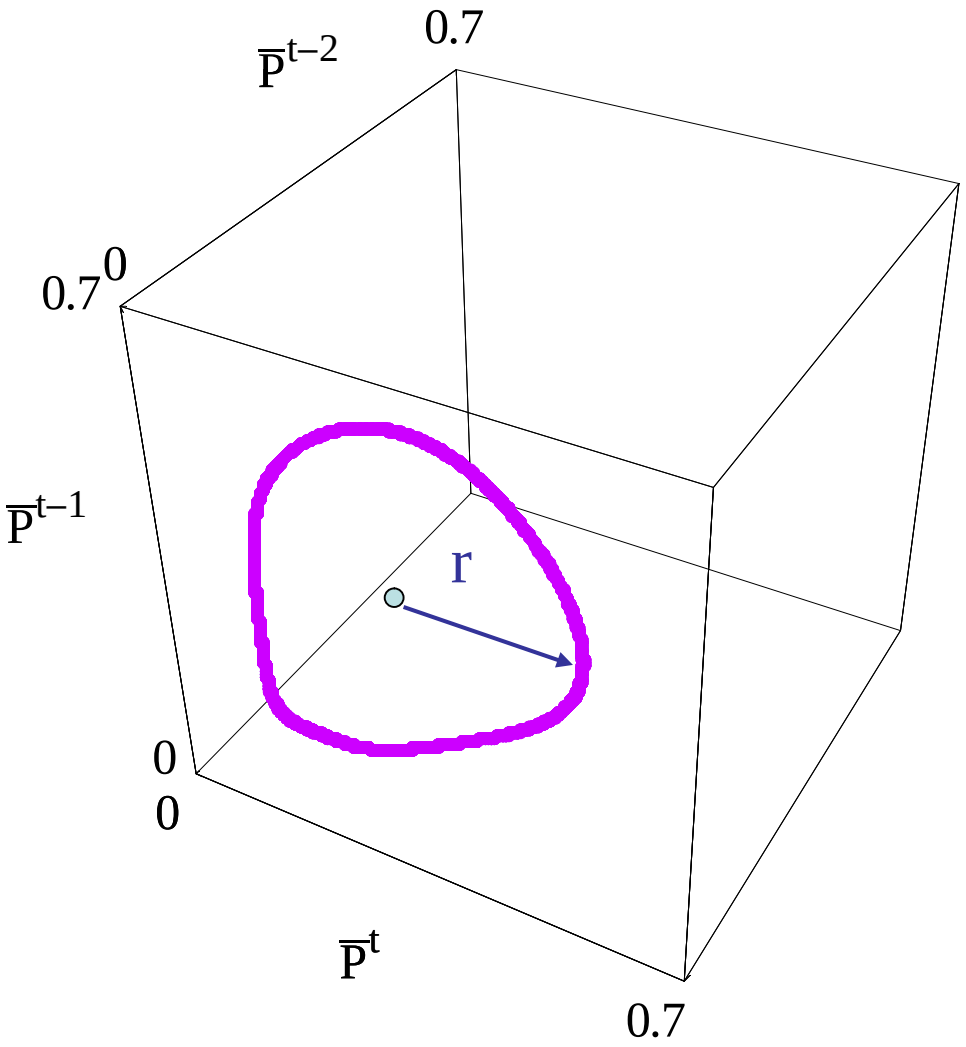
Eigenvalues in the Complex Plane, for increasing Noise Temperature , T=10

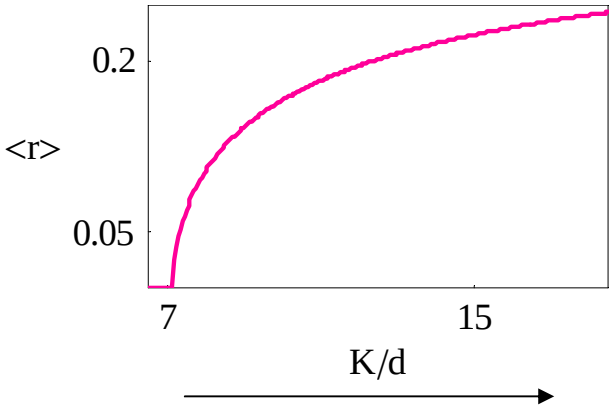


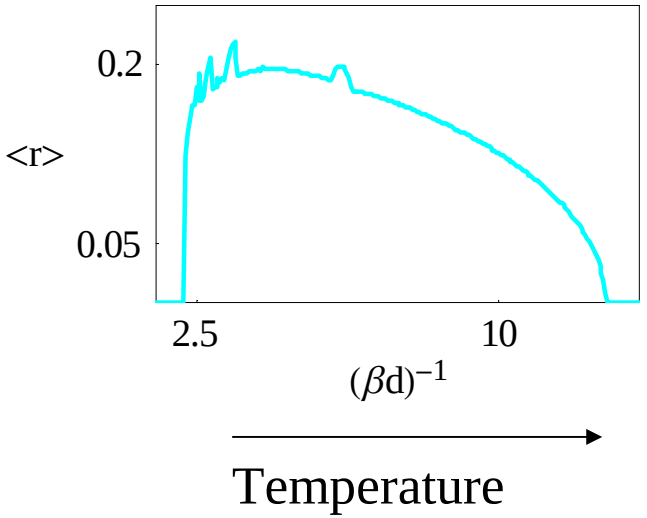
Phasediagram for T=10



Attractor for T=3



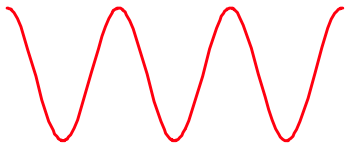




$$\langle r \rangle = \sqrt{\frac{1}{T_0}\sum_{t=1}^{T_0}(\overline{P}^t - \overline{P})^2}$$

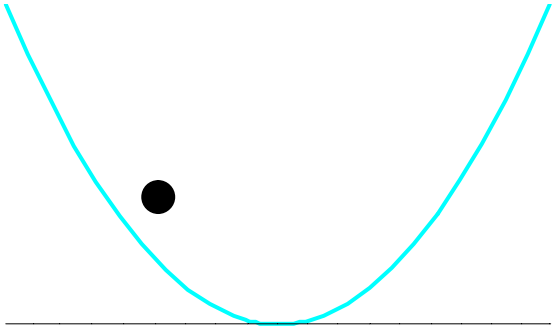
Control

Threshold

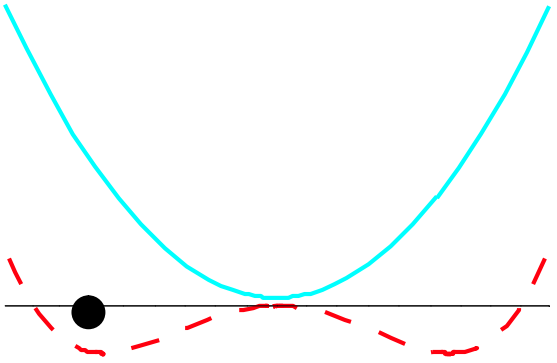


$$d = d_0 - a\cos^2(\omega_0 t)$$

$$d = -\frac{a}{2}cos(2\omega_0 t) + (d - \frac{a}{2})$$

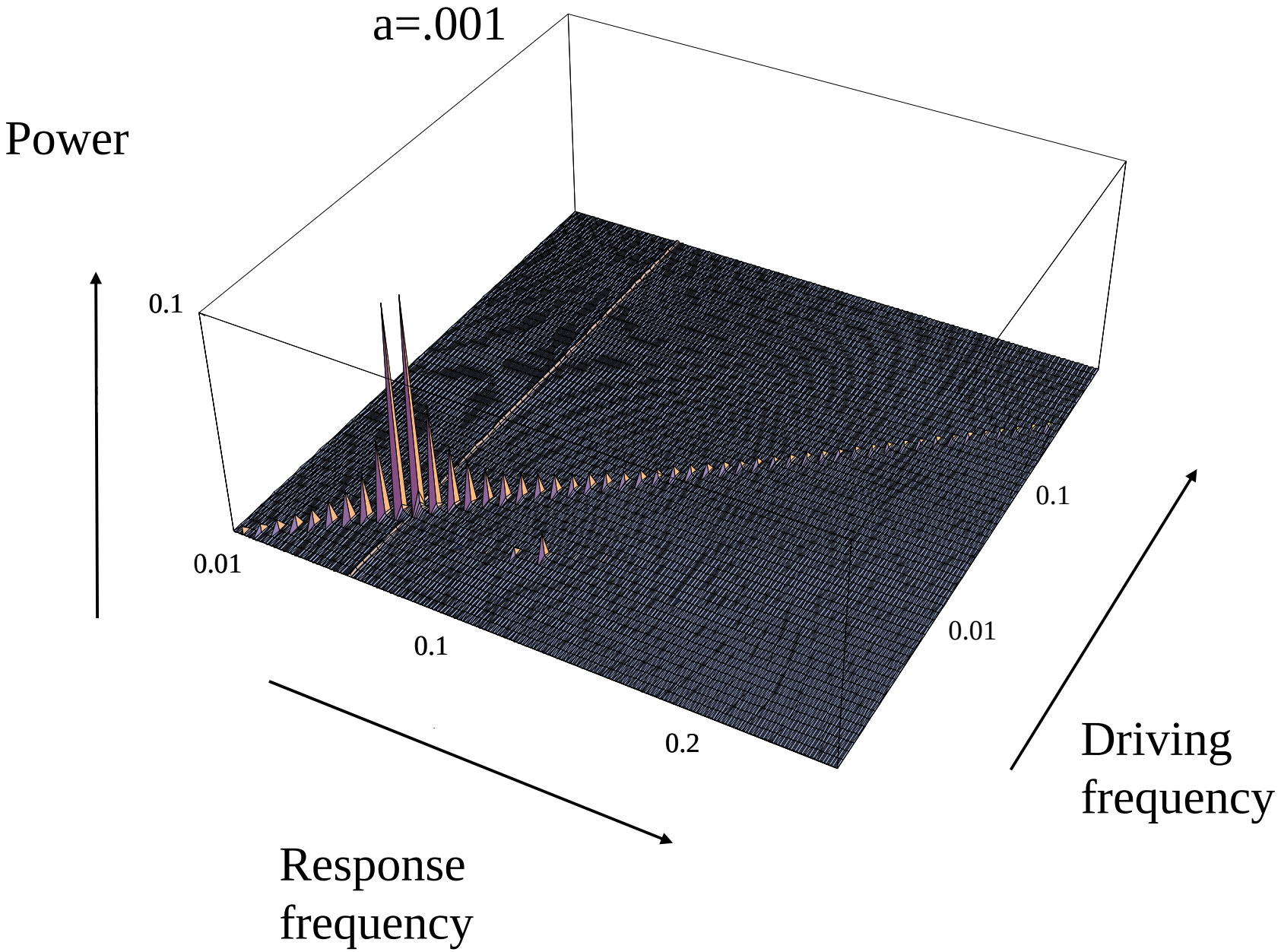


Resonance

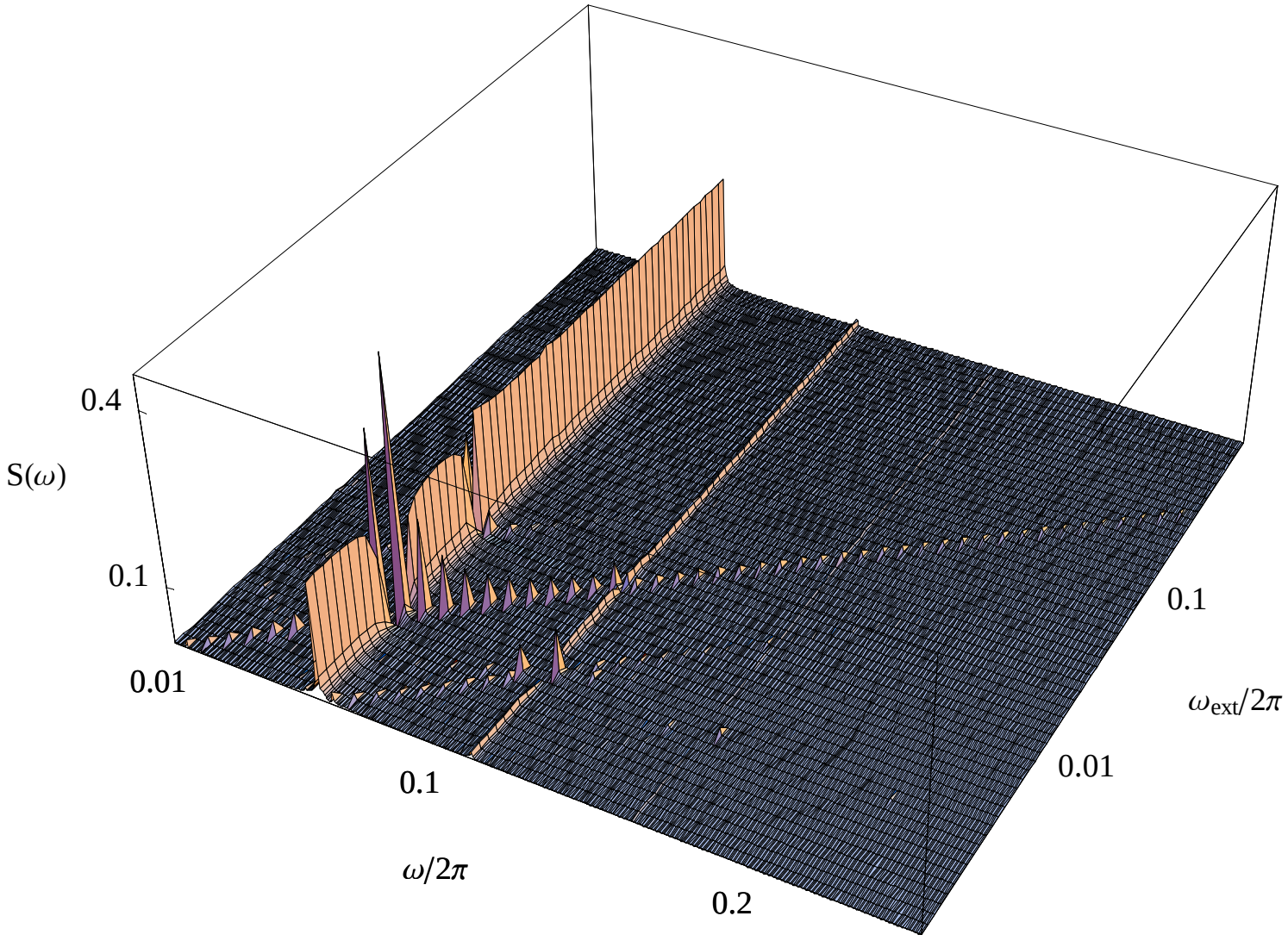


Instability

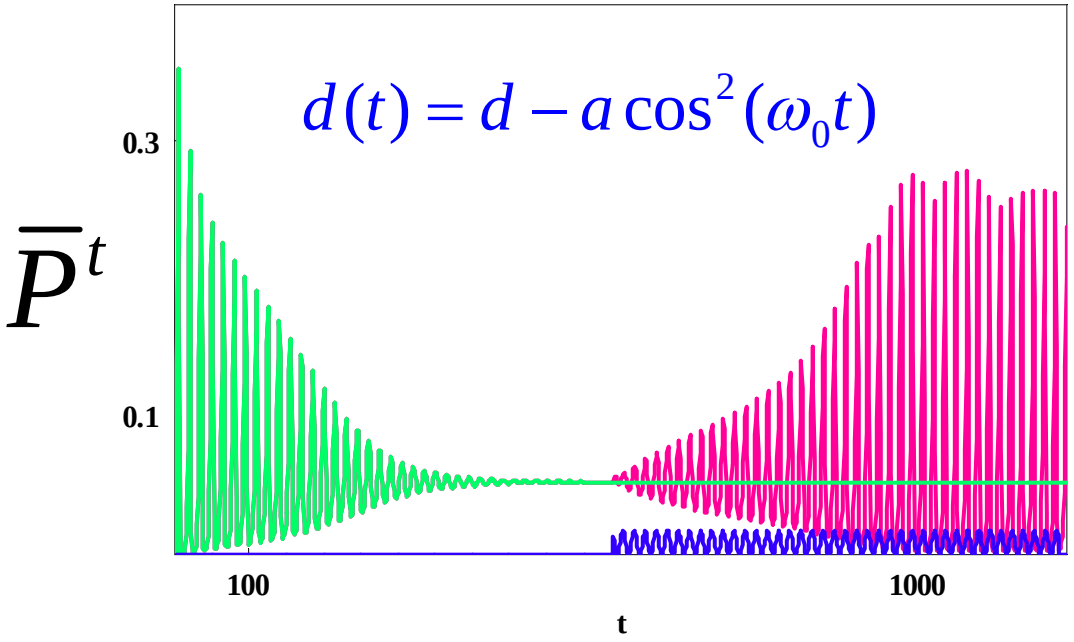
Resonance

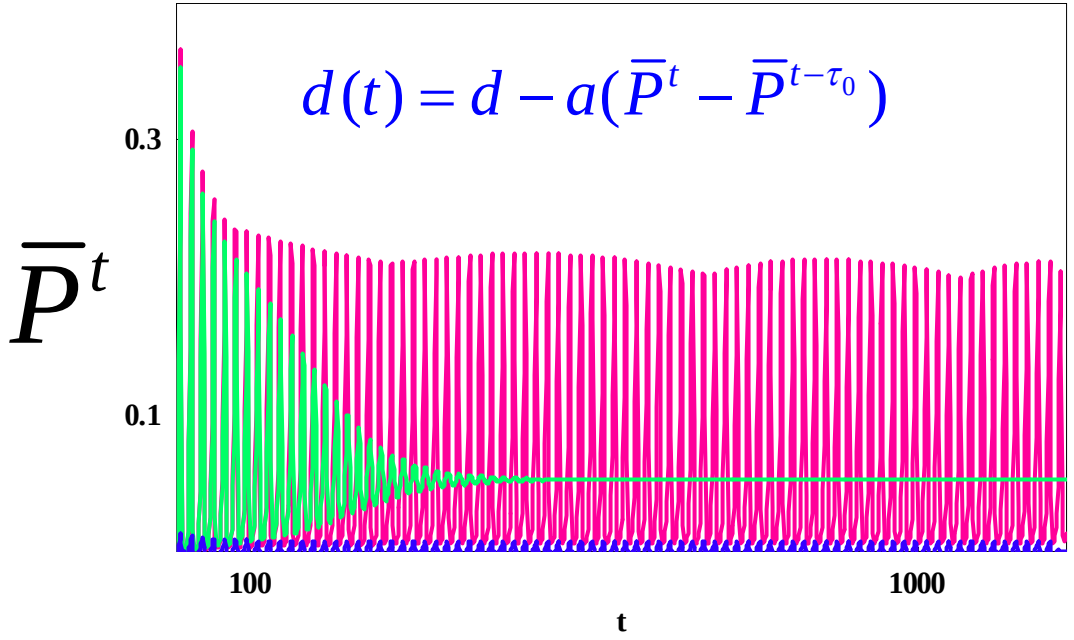

a=.001
Power
0.1
0.01
0.1
0.2
0.1
0.01
Driving
frequency
Response
frequency

Instability a=.005

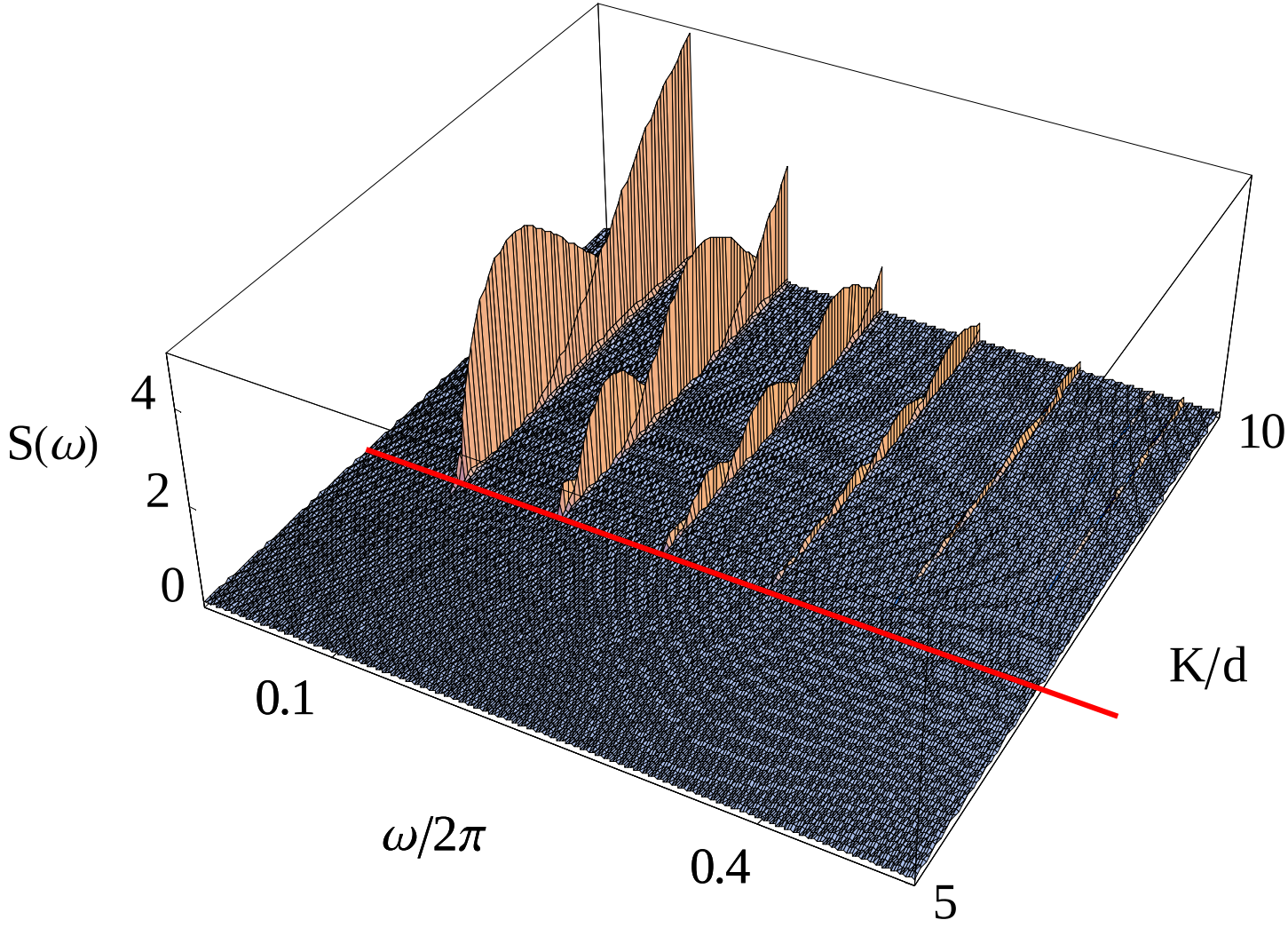


Control


$d(t) = d - a\cos^2(\omega_0 t)$
$\overline{P}^t$
0.3
0.1
100
1000
t


$d(t) = d - a(\overline{P}^t - \overline{P}^{t-\tau_0})$
$\overline{P}^t$
0.3
0.1
100
1000
t

Powerspectrum



Coupled threshold elements with selfinhibition show a phase transition to an oscillating synchronized state

This transition is very sensitive to external signals

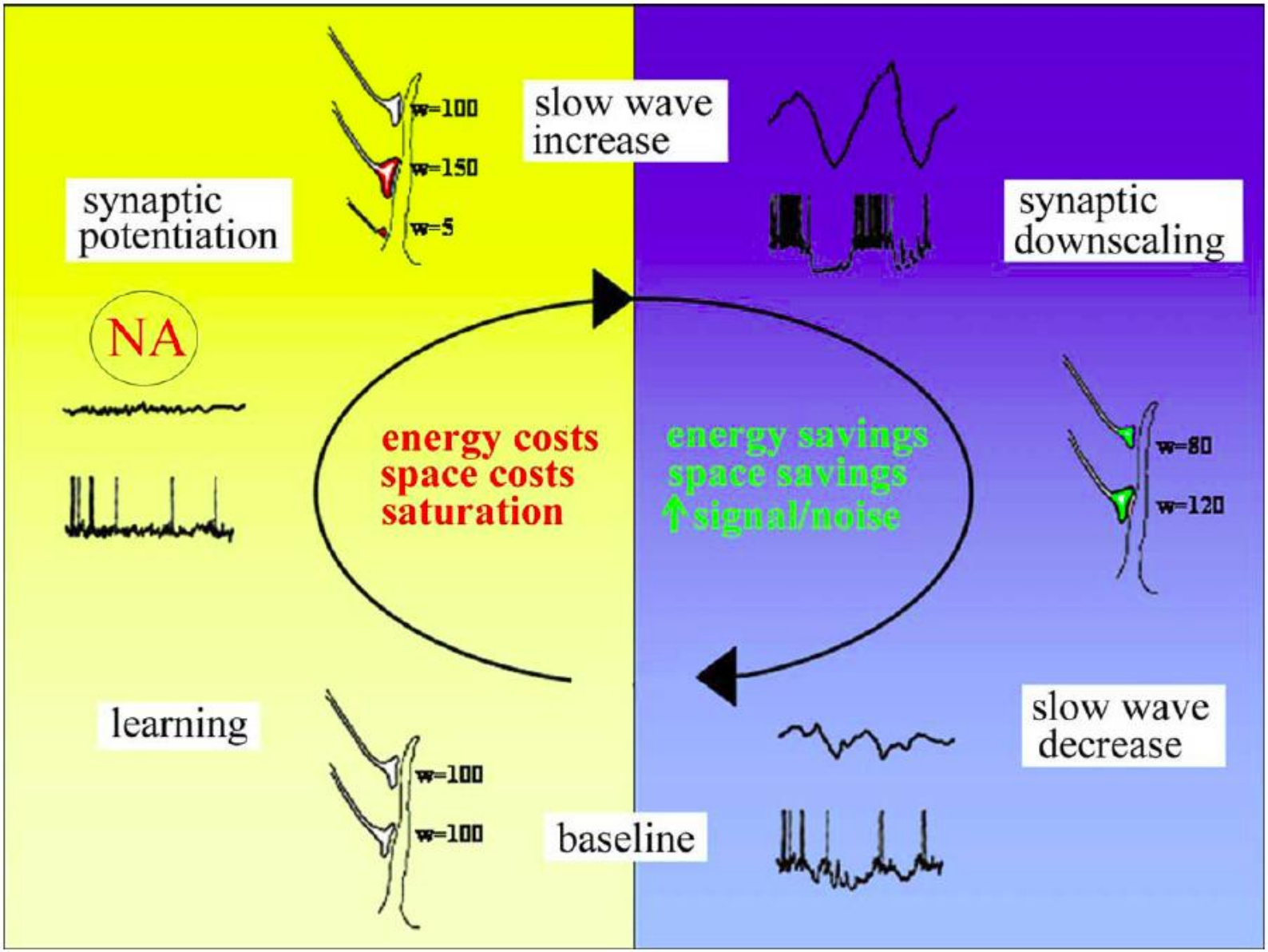
- Noise driven

- Control via oscillating fields acting on the threshold

  . frequency dependent resonances

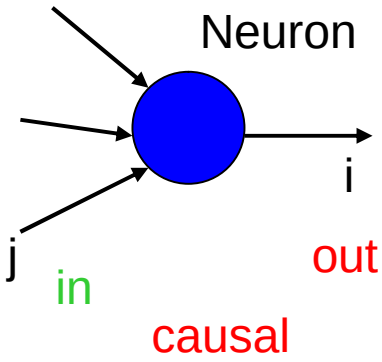
  . frequency independent induced transitions (abrupt)

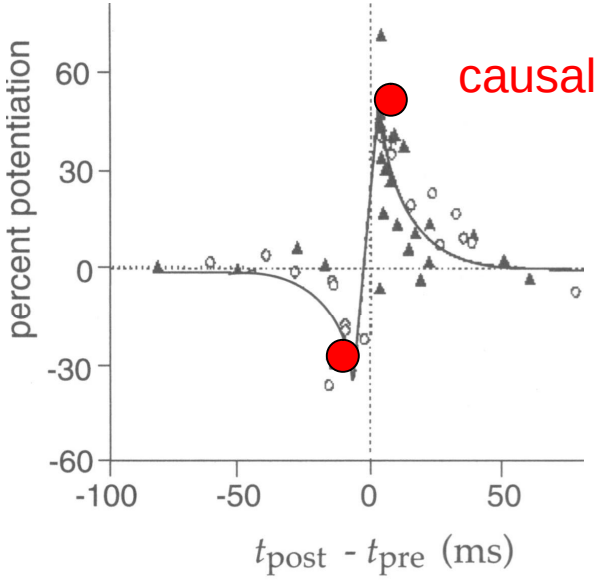
Control via Synaptic Change



Tononi and Cirelli Sleep Medicine Reviews **10**,49 (2006)

Synaptic change





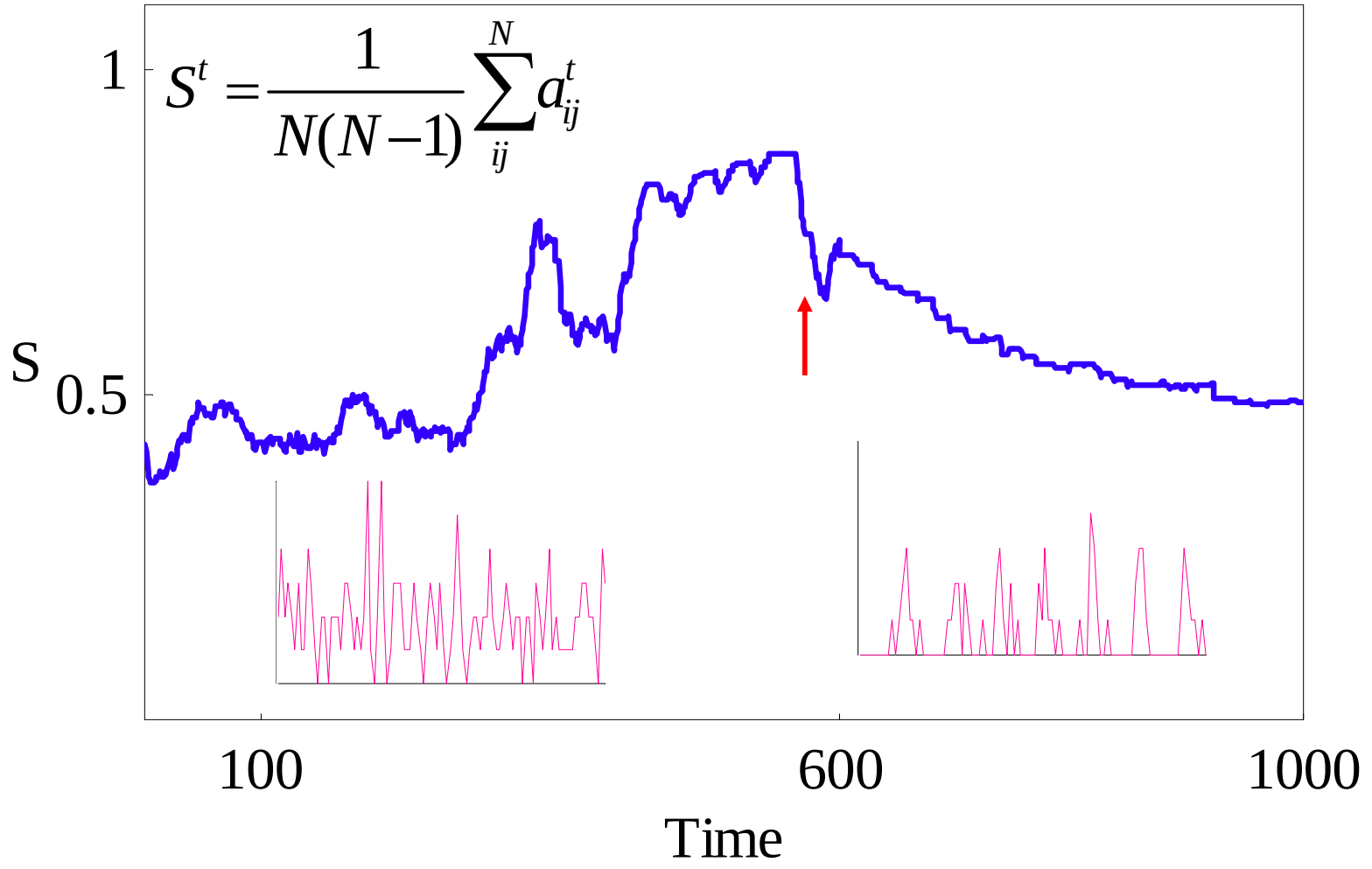
Model

$$a_{ij}^{t+1} = a_{ij}^{t} + \alpha x_i^{t+1} x_j^{t} - \beta x_i^{t} x_j^{t+1}$$

causal

$$0 \le a_{ij}^{t} \le 1.5 \qquad \alpha = 0.2 \qquad \beta = 0.25$$

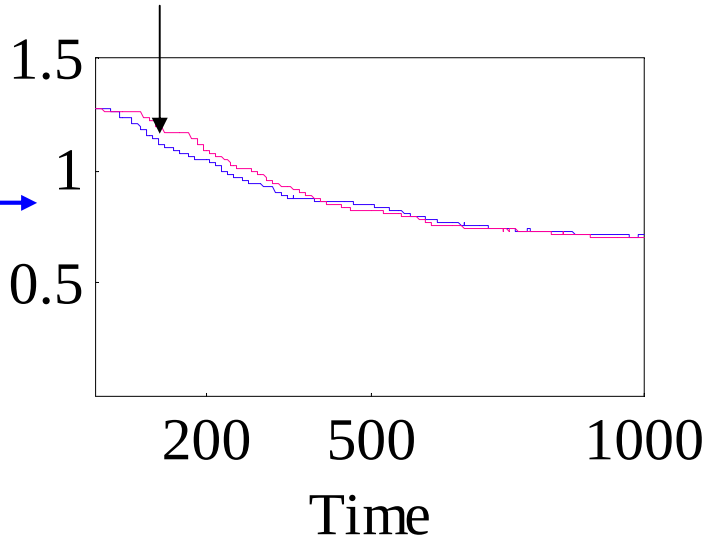
Mean Synaptic Strengths S



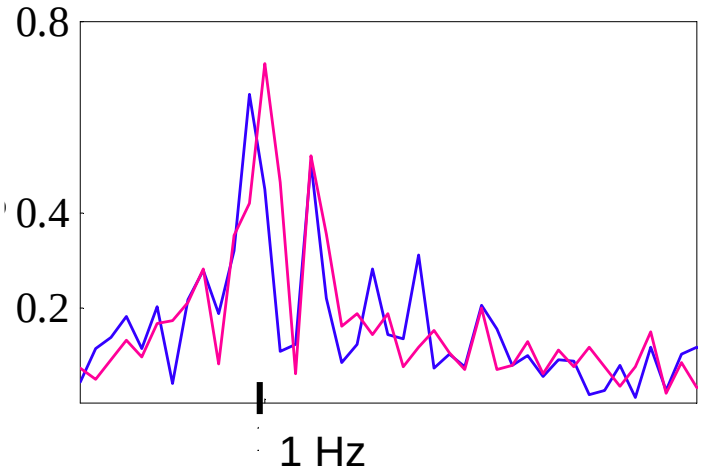
Mean synaptic strength

$$S^t = \frac{1}{N(N-1)} \sum_{ij}^{N} a_{ij}^t$$

External Stimulation

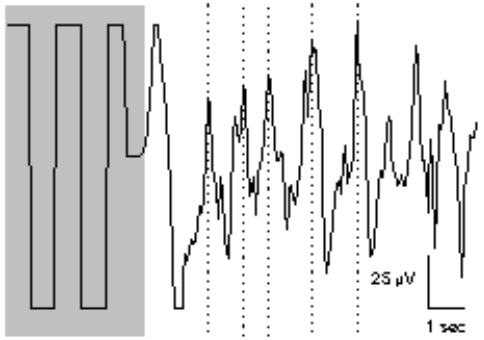


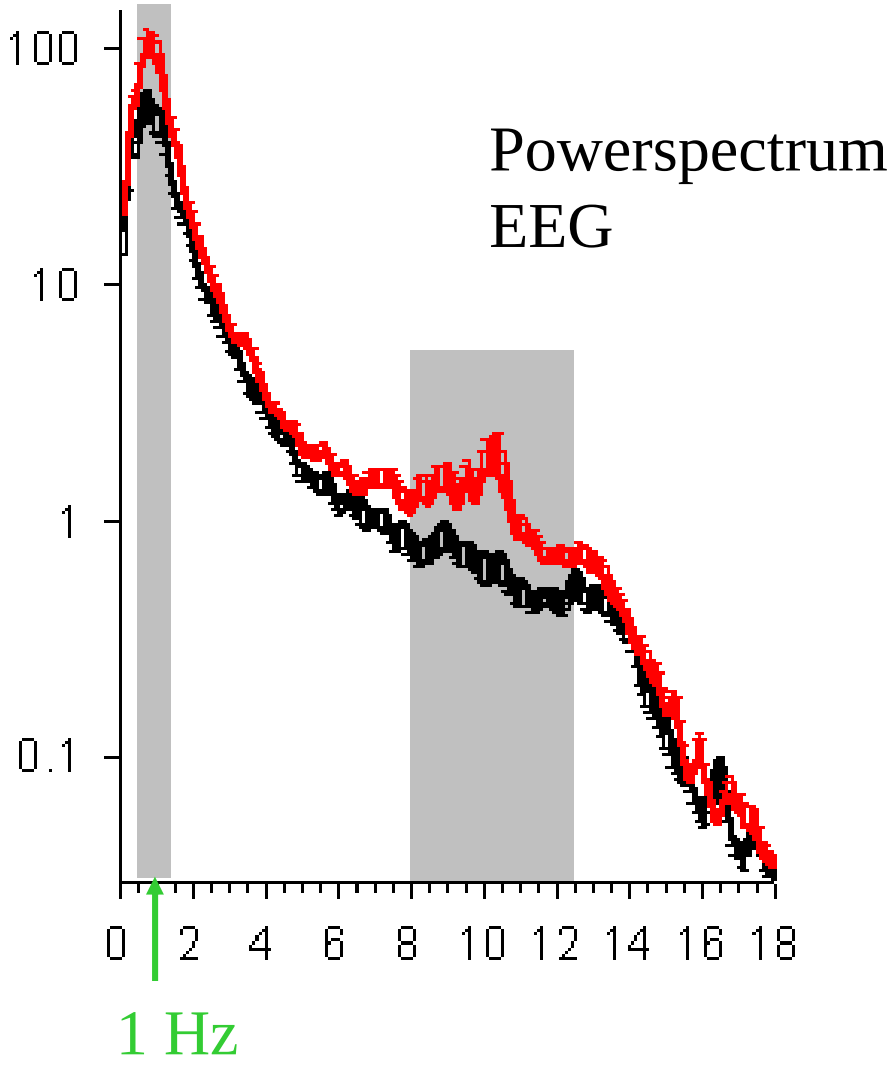
Power spectrum



Electrical

Stimulus





Powerspectrum

EEG

Nature (2006) 444, 610 **Boosting Sleep Slow Oscillations Potentiates Memory**

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J. C. Claussen, J. Mayer (Kiel)

M. Chavez , M. le van Quyen (Paris)

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  J. Mayer, H.G. Schuster, J.C. Claussen and M. Mölle,

  Phys. Rev. Lett. **99,** 068102 (2007)

- „Dynamical behavior and control of coupled threshhold elements with self inhibition“

  H.G. Schuster, M. Le van Quyen, M. Chavez, accepted

  International Journal of Bifurcation and Chaos**.**